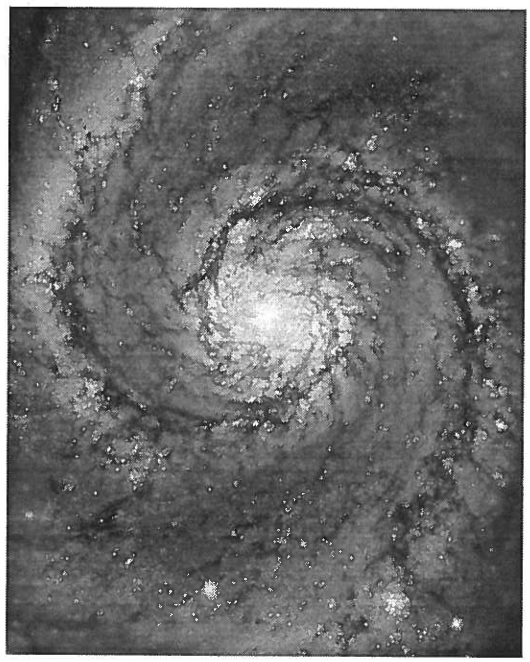
PCSE 499 Capstone Project Final Report

Modeling the Collapse of a Stellar Structure Into a Naturally Formed Spiral Galaxy



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Date: 1-May-2006

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Description of the problem:

The goal of our research is to model the collapse of a stellar structure such that a spiral structure forms naturally as a result of the collapse. Astrophysicists previously believed that material arms generated the spiral structure observed in spiral galaxies. However, material arms occur when all of the particles in the disk are moving together with differential rotation, meaning the particles near the center of mass are moving faster than the particles farther away from the center of mass. In this case the particles near the center are quickly winding around the galaxy until they are entirely wound up, resulting in the dissipation of the galactic structure. In reality, material arms only wrap around the galaxy once or twice before the galaxy is wound up and finally dissipates. On the other hand, spiral arms observed in galaxies in the universe have wrapped twenty to one-hundred times and still maintain a stable spiral structure. (Elmegreen 197) Therefore, spiral galaxies in nature do not contain material arms.

The current theory is that spiral density waves create the spiral structure seen in galaxies. A spiral density wave is a "gravitational perturbation that propagates through the disk."(Elmegreen 197) The density waves are moving faster than the surrounding particles. So they are moving through the particles at a speed that does not "wrap up as quickly as material arms," or at a speed that is moving too fast to wrap up at all.(Elmegreen 197)

We will begin our research with the search for a stable parameter space that will yield a region of stability, after a dense cloud has collapsed, stars have formed and the general shape of the galaxy has been established. The shape of the galaxy will be initialized with a spherical distribution of particles in the center and a less dense flat disk distribution of particles orbiting the sphere.

Initial preparations include familiarization with Linux Operating Systems, UNIX Commands, and Starlab software. Next we need to model a simple 2-body orbit with one body very heavy and the other body very light to analyze the result produced from Starlab and determine if it is consistent with 2-body systems observed in nature. If the result is reliable then we can continue with N-body spherical and disk distributions. A stable dense N-body spherical distribution and a stable less dense disk distribution will aid in the simulation of an N-body galaxy. The following research will be trying to obtain a stable galaxy. After a stable galaxy is obtained, we will add density perturbations in the shape of logarithmic spirals to see if the disk will collapse into a spiral galaxy observed in nature.

Plan for a solution:

To obtain a simple 2-body orbit we will implement a Kepler structure. A Kepler structure initializes a 2-body orbit that the user can manipulate to achieve varying results. Our purpose for implementing the Kepler structure will be to vary the masses of the 2-bodies such that 1 body is very heavy and the other body is very light. As we increase the mass of the heavy body we predict that the body will orbit closer to the center of mass and as we decrease the mass the body will orbit farther from the center of mass.

A stable spherical distribution can be implemented with either a Plummer or Sphere structure. The Plummer structure will initialize a non-homogenous N-body spherical distribution of particles; in which, the particles closer the center of mass are more dense than the particles farther away from the center of mass. The Sphere structure will initialize a homogenous N-body distribution of particles; in which, all of the particles in the orbit will have an equal mass. We predict that the Plummer structure will simulate a more stable spherical distribution because it initializes a sphere with a dense core, which is often observed in nature. However, in terms of stability a Sphere structure may still be capable of producing a stable orbit.

A stable disk distribution can be implemented with a Disk structure. The Disk structure will initialize a "near-keplerian disk, with N low mass objects orbiting a single massive object on almost circular paths." (Starlab Tools) This structure will determine the particles orbits by the very dense central mass, thereby initializing clusters of 2-body orbits.

After obtaining the parameter space for these structures we will need to run a Kira integrator to simulate the models. Kira is an N-body integrator with evolving hierarchical tree structure that allows the user to simulate the initial parameters over a user-specified time frame.

After obtaining a stable spherical distribution and disk distribution, we will attempt to model a stable galaxy. First we will obtain stable N-body spherical distribution with a dense core. Then we will obtain a less dense stable N-body disk distribution. We will run a merge_snaps command to join the simulations together and then run the Kira integrator to analyze the galaxy as it evolves.

If a stable galaxy can be formed we will attempt to set up an initial system with a dense core and a circular disk with density perturbations in the shape of logarithmic spirals with the appropriate perturbations of the tangential velocity of the disk particles. We will analyze the galaxy to see if these "density wave" arms are stable and if they reproduce the spiral structure seen in galaxies around the universe. Next we will set up a system with a dense core and a 'pressure wave' spiral arm to see if this structure reproduces a stable spiral galaxy. Furthermore, we will set up an initial "cloud" which will evolve into a system with a 'pressure wave' spiral arm structure.

In addition, all of the simulations after being run through the integrator will be displayed using an xstarplot command which will plot an N-body system and allow the user to view the evolution of the simulation in a 2 or 3 Dimensional reference plane and increase or decrease the speed at which the model simulates.

Implementation of the plan:

The simulation of the two body circular orbit was achieved by setting up a Kepler simulation (refer to Figure 1), which initializes a two body simulation. We assigned one body a very high mass density relative to the other body which had a low mass density. After simulating the procedure we obtained a stable two body circular orbit. The circular orbit consisted of a heavy body orbiting closer to the center of mass and the less dense body orbiting farther away from the center of mass. Also by changing the masses of the two bodies we achieved varying results in the simulation. First let's get familiar with Kepler's laws.

$$M_{CM} = M_A + M_B \quad \text{(Equation 1)}$$

Equation 1 shows that the center of mass is equal to the sum of the masses of the bodies in the system.

$$\overset{\varpi}{r}_{CM} = \frac{M_A}{M_A + M_B}\overset{\varpi}{r}_{AB} \quad \text{(Equation 2)}$$

Equation 2 shows the formula for computing the radii vector for the center of mass. If one body (M_A) is much larger than the other body (M_B), the heavy body would orbit very close to the center of mass.

$$\begin{aligned} &M_A >>> M_B \\ &M_{CM} \approx M_A \qquad \text{(Equation 3)} \\ &\overset{\varpi}{r}_{CM} \approx \frac{M_A}{\sim M_A}\overset{\varpi}{r}_{AB} \approx \overset{\varpi}{r}_{AB} \end{aligned}$$

Equation 3 shows that if one bodies mass is much larger that the other body, the larger body has the greater portion of mass for the system. Therefore, the larger body will orbit very close to the center of mass for the system. With the Kepler simulation the heavy body appeared to be motionless since it was so close to the center of mass. However, as long as there are two bodies in the system, we can increase the mass of the larger body to infinity and it will never be able to orbit exactly at the center of mass. If we held the mass of the less dense body constant, the more we decreased the mass of the larger body the farther away from the center of mass it orbited. When the two masses were equal, the two bodies orbited the center of mass in a 180 degree phase on equal radii.

$$M_A = M_B$$
$$\vec{r}_{CM} = \frac{M_A}{2M_A}\vec{r}_{AB} = \frac{1}{2}\vec{r}_{AB} \quad \text{(Equation 4)}$$

Equation 4 shows that if the masses of the two bodies are equal, the two bodies will be the same distance away from the center of mass. The results of this simulation were exactly as we predicted and proved that Starlab could successfully simulate a 2-body orbit. Now that we have produced a successful 2-body orbit we can begin to simulate more advanced models with Starlab.

The simulation of an N-body spherical distribution was achieved by initializing both a Plummer (refer to Figure 2-a) and Sphere (refer to Figure 3-a) structure and initiating a Kira integration tool that simulates the N-body structure over a user-specified period of time. After analyzing the Plummer and Sphere structures we found that both provided stable N-body orbits over prolonged periods of time. However, there were significant differences in the integration time between the two distributions.

The Plummer structure was our initial choice for creating a stable spherical distribution because it modeled a distribution of particles that is similar to spherical distributions in the universe. The Plummer structure initializes a non-homogenous N-body spherical distribution of particles. In this case, the particles near the center of mass are more dense than the particles farther away from the center of mass. Also in our parameter space, the user defined radius will only contain 60% of the enclosed mass because there is an exponential trailing off of matter as particles orbit farther from the center of mass. While we did find a stable Plummer simulation the Plummer structure took a significant amount of CPU time to calculate.

We found that the reason the Plummer structure integrated slower was because of the dense core initialized at the center of the sphere. The dense core contains the majority of the mass for the spherical distribution increasing the probability for close approaches. In fact, the dense core does produce a significant amount of close approaches. This means the particles are not feeling the average gravitational field, so the average gravitational field is a lot weaker. Also the individual masses of the particles are larger, so the individual pear-wise fields are stronger. In effect, the particles are moving under the influence of its nearest neighbor and not the average gravitational field. Consequently, the gravitational field is stronger and changes fast as the particles move. The rapid change in the gravitational field increases the integration error for a large time step. So to preserve calculation accuracy, the integration step size has to significantly decrease by a factor of 1,000 to 10,000. The small integration step size has to be maintained until the two particles are far apart again, and are back into the average gravitational field. The small integration step size significantly increases the amount of time the CPU calculates the simulation. We discovered that it takes too long to integrate an N-body spherical distribution of particles with the Plummer structure; hence, we began analyzing other spherical structures.

The Sphere structure was the only other structure available to us that implemented a spherical distribution of particles. The Sphere structure initialized a homogenous N-body spherical distribution of particles. In this case, every particle in the system has the same mass. Also, since there is no exponential trailing off of matter as the particles orbit farther from the center of mass, we can define an outer radius so that we know the exact dimensions of the sphere. In addition, the separation of particles is larger because they can orbit the sphere from any radius with respect to the center of mass within the user defined radii. The combination of being able to define an outer radius and a larger separation of particles reduce the need for small integration step sizes and decrease the number of close approaches.

Since there is no dense core in the Sphere structure, the individual particles have less mass than the particles in the Plummer structure. In effect, if a close approach does occur the gravitational fields are weaker. Also the particles can be treated as moving in the average gravitational field. Consequently, the average gravitational field does not change much in the particles orbit. Therefore, the integration step size can be large because the error is minimized. If a large integration step size can be maintained the calculation time of the simulation will significantly decrease.

While the Plummer structure is a better physical model and more accurately portrays a spherical distribution of particles that is observed in the universe, due to hardware and time constraints, it is too expensive to calculate. Thus, we used a less physical model, the Sphere structure to conserve CPU calculation time.

After analyzing the results from the spherical distributions we have found that smaller N-body systems appear to be more erratic than larger simulations. This is because by reducing the number of bodies in the system, the simulation compensates by increasing the mass of each individual body.

$$m = \frac{1}{n} \text{ (Equation 5)}$$

Equation 5 shows that the mass of each body is determined by one divided by the number of bodies in the system. The units of mass are in Solar masses.

$$1M_{SolarMass} = 1.989 * 10^{30} kg \text{ (Equation 6)}$$

Particles with increased mass have stronger gravitational forces that will attract other particles and then 'slingshot' them out of the system. This effect when combined with a more dense particle in a small N-body system will greatly perturb the overall orbit and stability of the system. If a larger body is ejected out of the system the entire system will lose a significant amount of its total mass and energy. In effect, the system will have trouble maintaining a stable orbit. However, in a larger N-body structure each particle has a smaller mass. If a small body is ejected out of the system, the system does not lose nearly as much mass or energy. In effect, the system maintains an increased chance that it can remain stable over prolonged periods of time.

Some of the results that failed were instances where the system collapsed on itself, also referred to as a 'core collapse'. This effect occurs because there is too much mass at the center of the sphere and the particles orbiting the center of mass do not have enough orbital velocity to remain in their orbit. In effect, the particles collapse to the center of the sphere perturbing the spherical orbit and the spherical distribution of particles is diminished.

Also there were instances where the particles were ejected out of the system. This effect is known as the 'slingshot effect'.(<u>refer to Figure 7-a</u>) In this case, particles were continuously ejected out of the system until the system either collapsed on itself or lost too many bodies to be considered a stable orbit. In a spherical distribution of particles where each particle has a large mass or there is a dense core, there is an increased probability of close encounters. When a close encounter between two massive particles occurs there is large energy transfer and increased gravitational force between the particles. In effect, one particle is ejected out of the system with most of the energy that was transferred, thereby, leaving the remaining particle with less energy. Now the remaining particle does not have enough energy or orbital velocity to remain in its orbit. Thus, the remaining particle collapses to the center of the sphere and begins to orbit in elliptical paths, where it now has an increased probability of having another close encounter. The end result is that particles increasingly get ejected out of the system until there is not enough energy to kick particles out. Thus, only a few particles remain in the system orbiting the center of mass.

The simulation of an N-body disk distribution (refer to Figure 4-a) was implemented through a Disk structure and initializing a Kira integration tool. The Disk distribution initialized a "near-keplerian disk, with N low-mass objects orbiting a single massive object on almost circular paths."(Starlab Tools) In effect the Disk structure initializes a massive particle at the center with a less dense disk of N-body particles orbiting the center. This structure was not as difficult to obtain stability because the particles were large enough and far enough apart to remain stable.

However there were some initial stability issues. The simulations varied in results based on the relationship between the total mass of the disk and the mass of the central object, the inner and outer radii, and the number of bodies used in the simulation. In earlier results the disk appeared to be collapsing on the center of mass but would then quickly expand outward. In this case, the center particle was initialized with too much mass, thus the particles in the disk did not have enough orbital velocity to maintain in their orbit. In effect, the disk would collapse. A successful stable orbit was obtained with a 500 body system while only losing a few of the bodies in a prolonged time frame.

A galactic structure was obtained by combining an N-body circular disk with a high density core and a low density disk with an N-body Sphere, both positioned around the same center of mass. In order to create the galaxy we had to populate the disk distribution and the spherical distribution separately. Afterwards we merged the disk distribution and the spherical distribution structures together and integrated the distributions collectively.

The results were usually the disk and the sphere both collapsing on the center of mass. After the collapse several other results occur. Sometimes the system loses most of the bodies, but a smaller subset of bodies remains orbiting around the same center of mass while moving off in a random relative direction. Another common result was that the system collapses and all the bodies randomly exit their orbit.

After analyzing the effects we determined the problem occurs because we are forced to generate the disk distribution and the spherical distribution separately. For example, when we initialize the disk, it contains a dense center particle with a mass of 50 solar masses (to be extreme). Then we put a sphere with a total mass of 1 solar mass in the center. So we effectively changed the mass of the central body of the disk from 50 to 51 solar masses. This should have no apparent effect on the disk. However, the sphere's total mass changed from 1 to 51 solar masses. This is a massive difference resulting in a significant collapse of the sphere.

After all attempts at creating a stable galaxy were unsuccessful we decided to scale the distributions before integrating them with Kira. In almost every simulation the same result occurred, there was a spherical bouncing effect with the spherical distribution and the disk was ejected off of the sphere.(refer to Figure 6) The system appeared to be stable for a prolonged period of time in the beginning but after further analysis and increased simulation time frames, the sphere consistently fluctuated as if it were collapsing and expanding and the disk slowly but continuously expanded until it was ejected out of the system.

Scaling our system effectively took the mass at the center body and distributed it equally to all of the particles in the system. Thus, the center of mass becomes less dense. With a less dense center of mass the sphere will not entirely collapse. Instead, the result is a spherical bouncing effect and the disk being ejected out of the system. This occurs because the total mass of the central body for the disk is less dense than when the disk was initialized. Therefore, the disk particles will have more kinetic energy, pushing their orbits farther from the center body. Also the sphere distribution is perturbed by the disk and the slight increase in total mass, so it will continue to fluctuate. The end result, is that the two bodies continuously perturb each other, and in this case the sphere remains semi-stale while the disk is ejected out of the system.

In all, we obtained one potential stable galaxy which will take further analysis to officially determine its stability.(refer to Figure 5-a) The reason we are unable to determine the stability thus far is because in every case that we have simulated it, the computer crashes and stops integrating at exactly 725 time frames. With 725 time frames of data, this galaxy simulation is far more stable than any other simulation produced and shows signs of prolonged stability. In addition, there is significant differential rotation; in which, the particles near the center of mass are moving faster than the particle farther away from the center of mass, and there is signs of spiral rotation. This simulation not only shows promising results for producing a stable galaxy but also shows signs of producing a spiral galaxy.

Design alternatives and constraints:

The entire design of this research project was accomplished with no budget. We obtained an aged computer with a Linux Operating System from Christopher Newport University and downloaded the Starlab software from the Internet free of charge under the GNU License agreement. However, if there was funding for the research, we would have purchased a faster computer with a Linux operating system. A dual Pentium IV processor CPU would have significantly decreased the amount of CPU time depleted in integrating the simulations. In addition, it would allow us to run multiple simulations consecutively, increasing the amount of work that can be accomplished and analyzed.

Evaluation and conclusions:

We successfully obtained stable 2-Body orbits, N-body spherical distributions, and N-Body disk distributions. Also in search for a stable galactic parameter space we have potentially, pending further analysis, found a region of stability that could result in a stable galaxy and aid in the production of a spiral galaxy. Now that a stable galaxy has potentially been simulated, future research should begin with completing the simulation of a stable galaxy. Then add density perturbations to the disk in the shape of logarithmic spirals. Analyze the model to see if the perturbations result in density wave arms, producing spirals around the galaxy, and if the galaxy resembles the spiral structure seen in nature. If this is successful then we need to model a dense cloud; such that, it will evolve into a galaxy with a dense core and a 'pressure wave' spiral arm structure.

Appendices (A)

How to reproduce my research results:

1. Set up a stable 2-body orbit with 1-body very heavy and the other body very light:
   a. kepler -m 100 -m2 1 -d .01 -O yes -t 50 -T 50 > [user-specified output filename]
2. Set up a stable N-body spherical distribution:
   a. makeplummer -n 250 -i -u | kira -d 1 -D 1 -h 1 -t 600 > [user-specified output filename]
   b. makesphere -n 500 -i -R 25 -u | kira -d 1 -D 1 -h 1 -t 100 > [user-specified output filename]
3. Set up a stable N-body disk distribution:
   a. makedisk -n 500 -i -r 150 -R 450 -m 100 -M 1000 | kira -d 1 -D 1 -h 1 -t 600 > [user-specified output filename]
4. Set up a stable galaxy:
   a. makesphere -n 128 -i -R 10 -u > [user-specified output filename 1]
   b. makedisk -n 127 -i -r 11 -R 25 -m 5 -M 10 > [user-specified output filename 2]
   c. cat [filename 1] [filename 2] | merge_snaps > [user-specified output filename 3]
   d. scale < [filename 3] > [user-specified output filename 4]

e. kira -d 1 -D 1 -h 1 -t 5000 < [filename 5] > [user-specified output filename 6]

5. Set up an unstable galaxy:

a. makesphere -n 128 -i -R 50 -u > [user-specified output filename 1]

b. makedisk -n 127 -i -r 60 -R 185 -m 1 -M 2 > [user-specified output filename 2]

c. cat [filename 1] [filename 2] | merge_snaps > [user-specified output filename 3]

d. scale -s < [filename 3] > [user-specified output filename 4]

e. kira -d 1 -D 1 -h 1 -t 5000 < [filename 5] > [user-specified output filename 6]

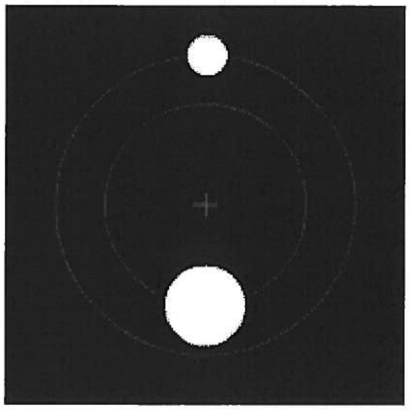
6. Display the simulations in a plot:

a. xstarplot -d [3 or 2] < [user-specified input filename]

Appendices (B)

Simulation Results:

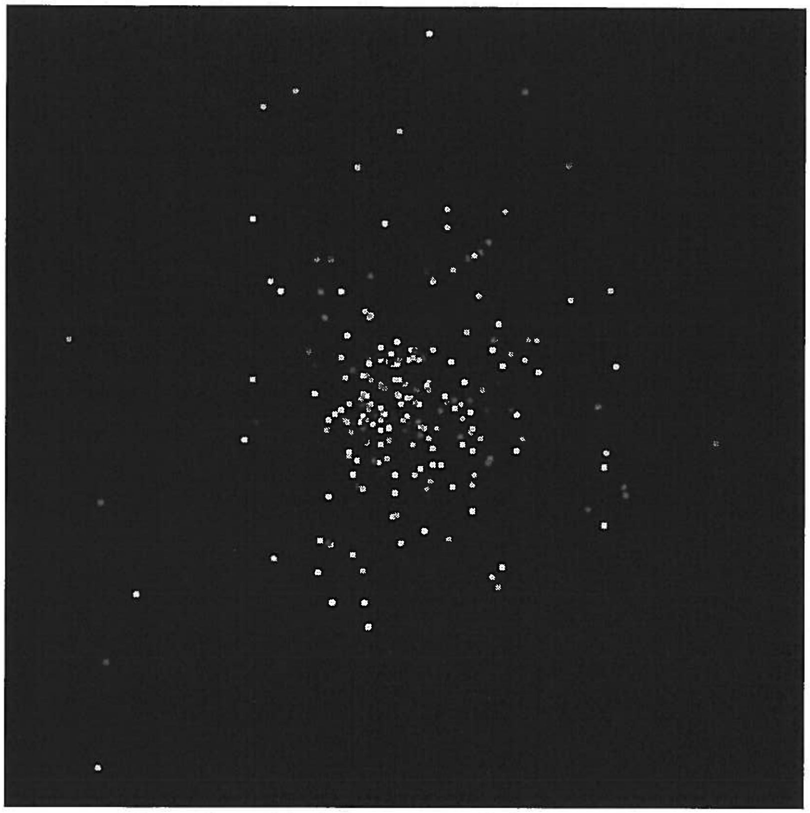
2-body Kepler Orbit (Figure 1):



(Wikipedia)

<Click Here to Open as a Simulation>

128-body Plummer Distribution - Slow (Figure 2-a):



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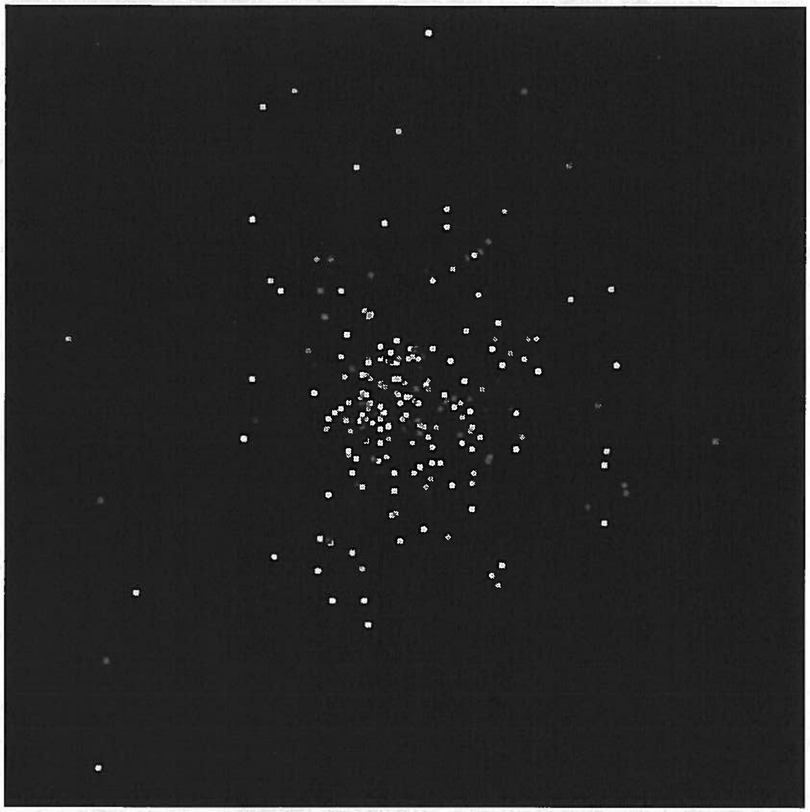
128-body Plummer Distribution - Normal (Figure 2-b):



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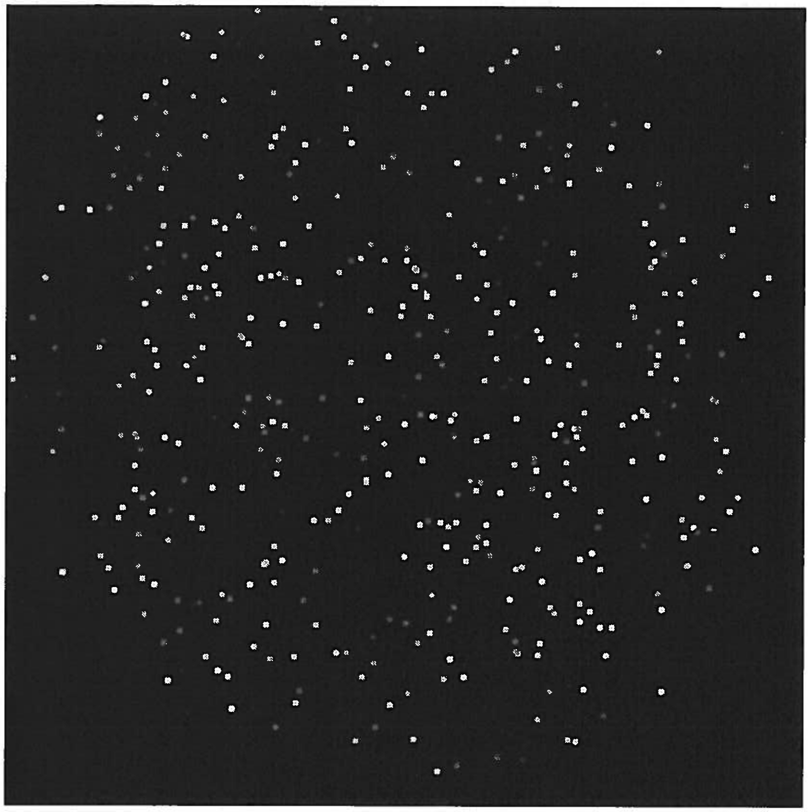
256-body Sphere Distribution - Slow (Figure 3-a):



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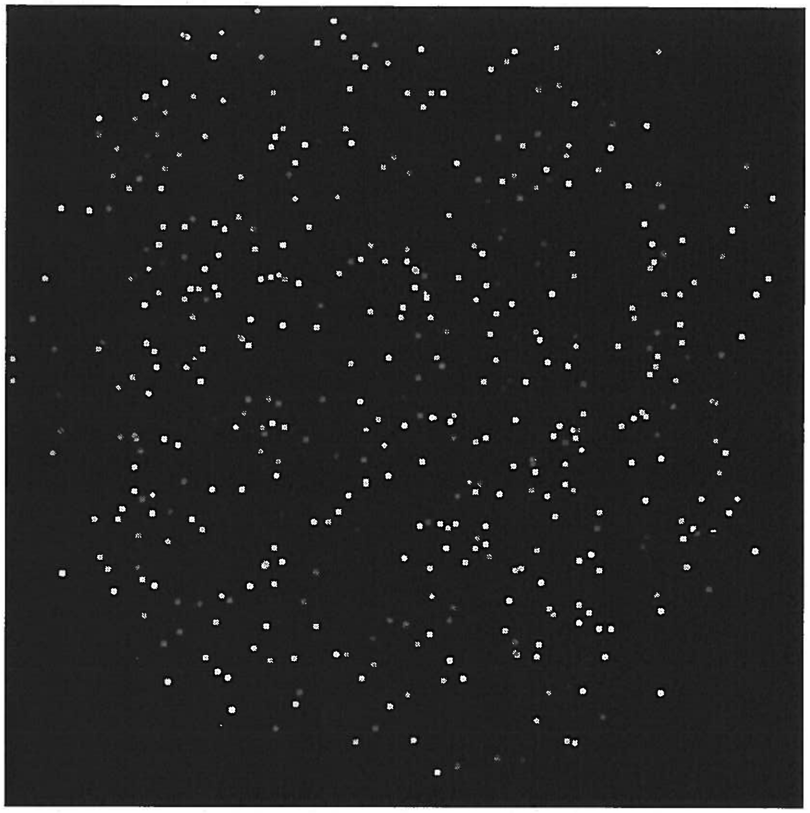
256-body Sphere Distribution - Normal (Figure 3-b):



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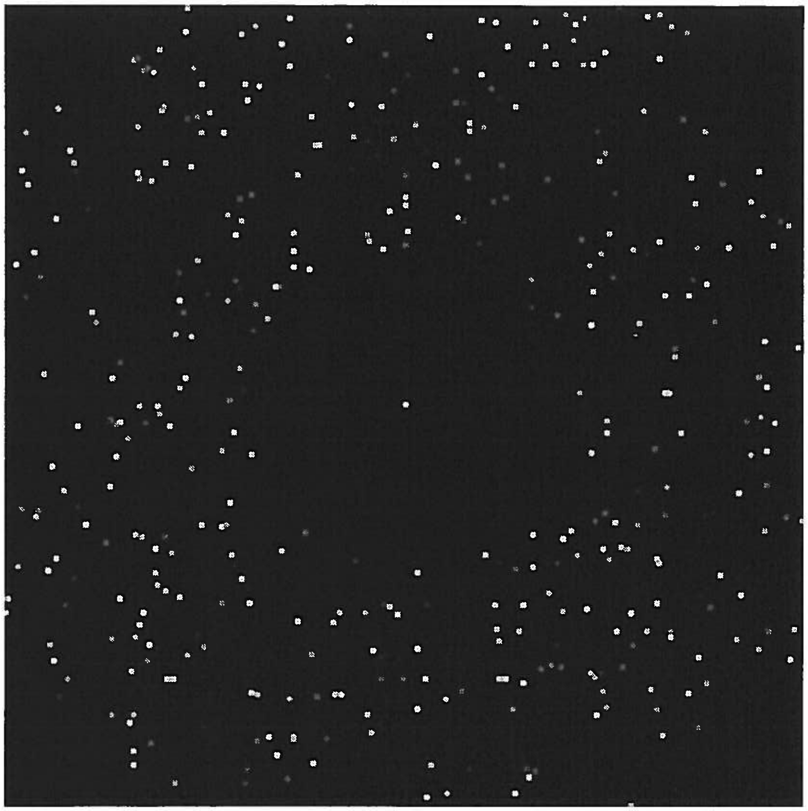
256-body Disk Distribution - Slow (Figure 4-a):



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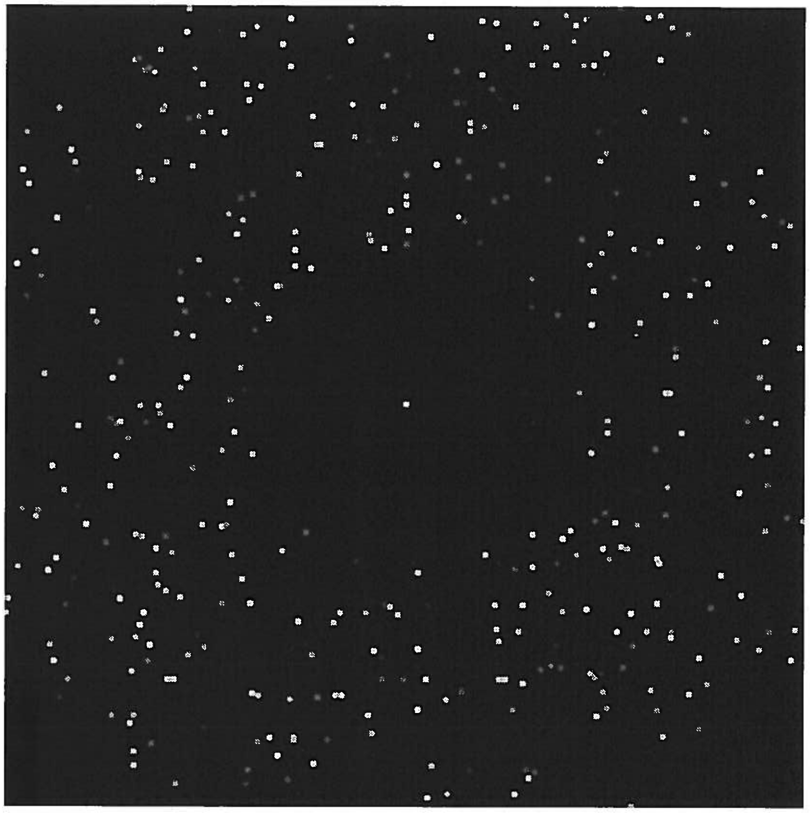
256-body Disk Distribution - Normal (Figure 4-b):



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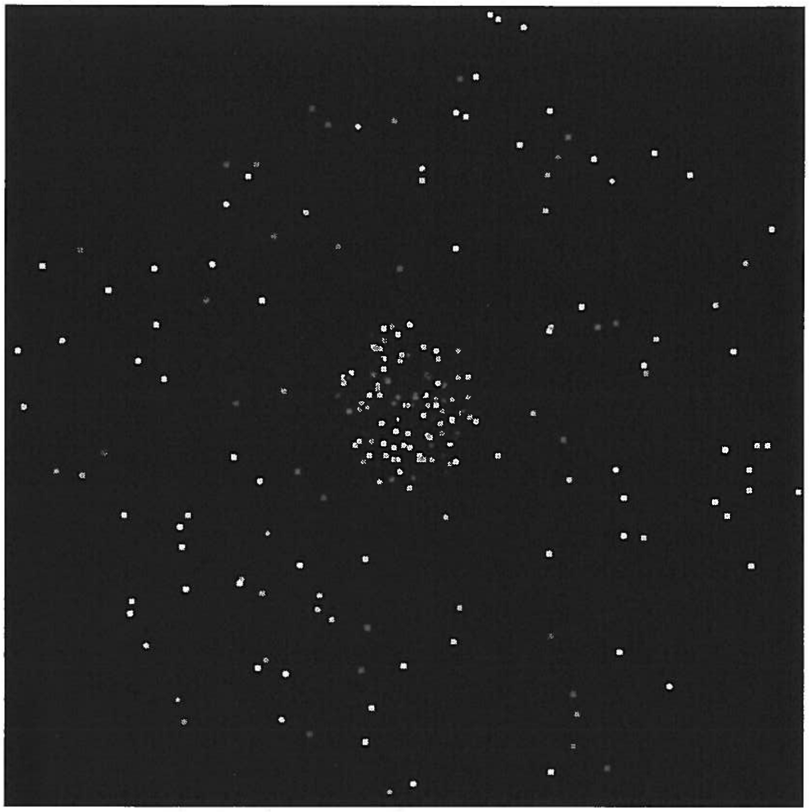
256-body Stable Galaxy - Slow (Figure 5-a):



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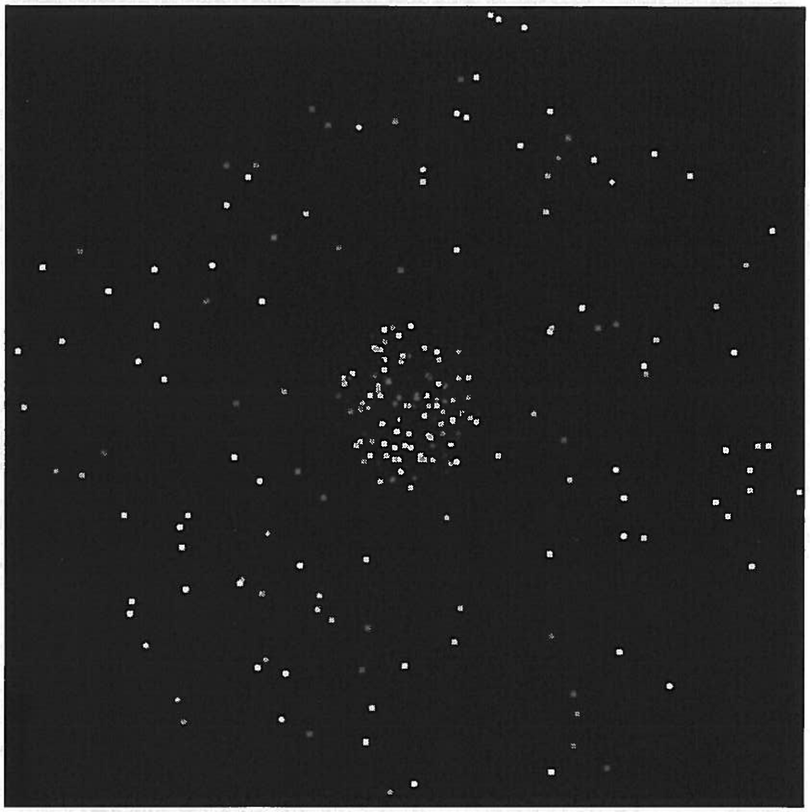
256-body Stable Galaxy - Normal (Figure 5-b):



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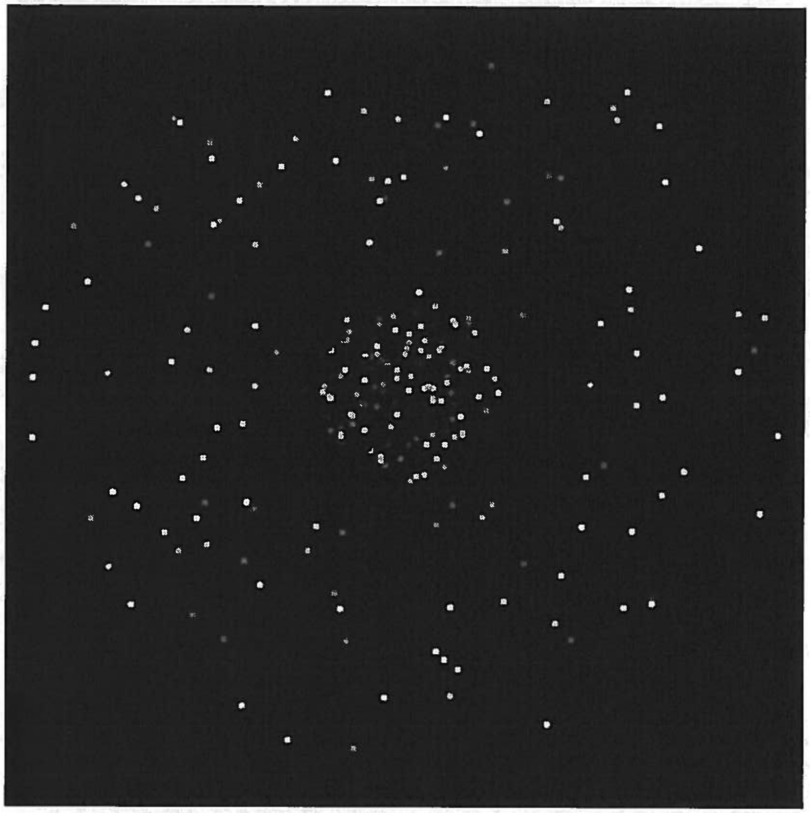
256-body Unstable Galaxy - Normal (Figure 6):



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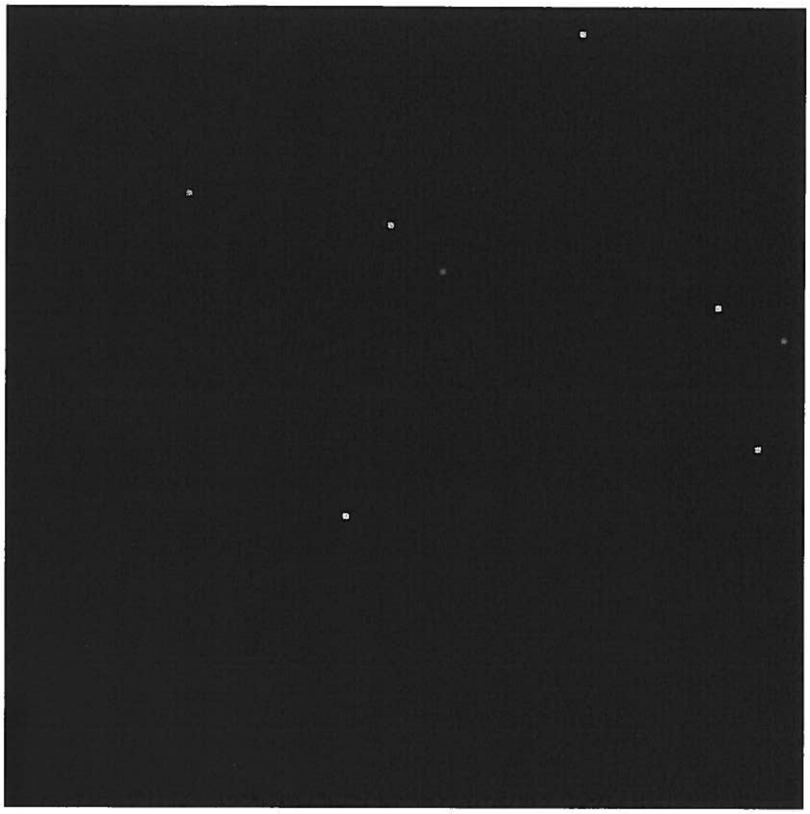
10-body Sling-shot Effect - Slow (Figure 7-a):



<Click Here to Open as Simulation>

10-body Sling-shot Effect - Normal (Figure 7-b):

<Click Here to Open as Simulation>

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