Final Report: Study on the Effect of Mach Angle on Delta-wing Angle efficiency through simulations in Kerbal Space Program

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1 Introduction

Delta-winged aircraft have existed since the mid-1500s (1) and reached prominence in the jet age during the 1950s. Delta-winged aircraft exist in many variations but are easy to identify due to their triangular shape. Delta-winged aircraft are "the top tier for great maneuverability and satisfactory take-off and landing speeds"(2) and are most often used by jets exceeding speeds of Mach 1. Delta-winged aircraft come in many different design variations and angles. If a delta wing is formed by a right triangle with one leg parallel to the fuselage and one perpendicular to the fuselage, the delta wing angle is the angle closest to the nose of the aircraft.

A Mach number is a quantity that represents the velocity of fluid flow divided by the local speed of sound, with Mach 1 being 343 meters per second at sea level. Once a moving body exceeds Mach 1, it becomes supersonic and a Mach Angle is formed. A Mach Angle is the angle of deflection for the fluid as a supersonic body moves through the fluid. In aviation, a Mach Angle is the angle air is deflected by a supersonic or hypersonic aircraft.

Despite the intense research into both Delta-winged aircraft and Mach Angles, the relationship between the Mach Angle and a delta-wing angle has been seldom studied. There is a substantial amount of research into the aircraft aerodynamic characteristic of a delta-winged (3) and the airflow over delta wings at subsonic and supersonic speeds (4). But there seems to be a lack of research that focuses on the angle of a pair of delta wings and its relation to a Mach Angle. Jet technology has been evolving since the 1940s and the perfection of supersonic air travel would bring the world closer together by making intercontinental trips take only a fraction of the time they do now.

The goal of this project is to investigate the relationship between Mach angle and the efficiency of a delta wing angle. This aerodynamic data will be found in the rocketry and aviation simulation software Kerbal Space Program. I will find the most efficient delta wing angle at multiple altitudes, and the most efficient angle overall.

2 Theory

I predict that as the Mach Angles become smaller, delta-winged aircraft with larger angles will become less efficient and there will be greater delta-V losses and the lift/drag ratio will decrease. At higher Mach Angles the airflow will be closer to the fuselage and there will be less airflow over the more blocky high-angle delta wings. Mach Angle is defined as:

$$\mu = arcsin(a/v) = arcsin(1/M) \tag{1}$$

where a is the speed of sound at some altitude, v is the velocity of a high-speed object, and M is the Mach number of that object. As the Mach number increases, either by a supersonic body moving faster or a decreasing speed of sound at some altitude, the Mach angle shrinks.

The drag force experienced by the jet is defined by:

$$Fd = Cd * 1/2 * \rho v^2 * A \tag{2}$$

Force of drag = drag coefficient times 1/2 the density of a fluid * fluid flow velocity squared * frontal area of a body. The drag coefficient quantifies the resistance an object faces in a fluid environment. Rho is the density of the fluid. v is the velocity of the fluid moving around a body. Frontal area of a body is the area that is in contact in the fluid.

The lift experienced by the jet is defined by:

$$FL = CL * 1/2 * \rho v^2 * A \tag{3}$$

Force of lift = lift coefficient times 1/2 the density of a fluid * fluid flow velocity squared * frontal area of a body. The lift coefficient quantifies the lift generated by an airfoil due to fluid deflection.

$$Lift - to - drag - ratio = Cl/Cd \tag{4}$$

Where Cl is the coefficient of lift and Cd is the coefficient of drag. The lift coefficient quantifies the lift generated when an airfoil is surrounded by a fluid. To find the estimated max of this ratio at subsonic speeds

$$Estimated(L/D)max = 4(M+3)/M \tag{5}$$

Where M is Mach number and (L/D)max is the maximum Lift/Drag ratio at Mach Numbers greater than 1.

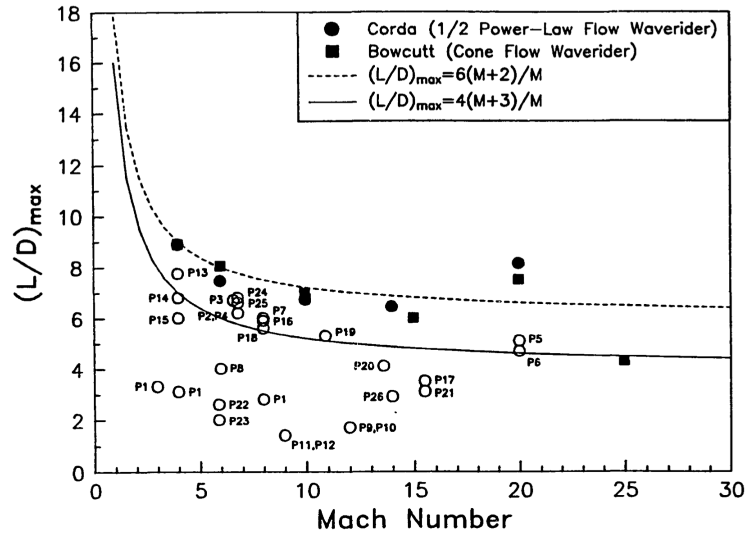

Fig. 1.1 - Maximum lift-to-drag ratio comparison for various hypersonic configurations.

Figure 1: Excerpt from "Viscous optimized hypersonic waveriders designed from flows over cones and minimum drag bodies", Stephen Corda, University of Maryland, 1988.

From this graph (5) we can derive the expected relationship between the Mach number of my simulated aircraft and the generated lift-to-drag ratio. Though Corda investigated Hypersonic waveriders, the supersonic relationship I'm looking for can still be found. There is a negative exponential relationship between Mach number and lift-to-drag ratio.

Delta V will be defined using the Tsiolkovsky rocket equation.

$$\Delta V = Isp * g * ln[mi/mf] \tag{6}$$

Where Delta V is impulse per unit mass, Isp is the specific impulse of an engine, g is the Earth's gravitational acceleration constant, mi is the initial or wet mass, and mf is the final or dry mass of the aircraft.

3 Methods

This first step of this project was conducted entirely inside KSP. Mods such as CKAN mod manager, B9 Aerospace Procedural Wings, Click Through Blocker, Ferram Aerospace Research Continued, Harmony 2, KSP Community Fixes, KSP Burst, Modular Flight Integrator, Module Manager, Patch Manager, Procedural Parts, Textures Unlimited, Toolbar Controller, TweakScale Redistributable, and Zero MiniAVC. Once this was done the next phase was to move on to aircraft design.

Early in the project the range of altitudes had been decided, so the test aircraft was designed to operate at altitudes ranging from 500 meters above sea level to 25,000 meters above sea level. The aircraft design went through many iterations and changes in the beginning to accomplish this. Initially, the test vehicle was going to be much smaller and flight data would have been collected by attaching the test vehicle to a launch vehicle, flying to the desired test altitude, disconnecting the test vehicle from the launch vehicle, and then finally collecting data. However, due to the Unnecessary complexity, this would have brought to the project, drop tanks were added instead. Both of these additions serve the same purpose, to give the aircraft the same amount of fuel when it was time to conduct a test, but drop tanks proved to be much simpler.

Next, this aircraft design was copied and 10 different versions were created. These 10 aircraft were equipped with differently angled delta wings at values of 90°, 80°, 70°, 60°, 50°, 45°, 40°, 30°, 20°, and 10°. These 10 different wing variations were fitted with the external fuel tanks. The 20, 45, and 90-degree variations are shown below.

Figure 2: Left, Didact VII 20 degree variation with drop tanks as viewed from the side.

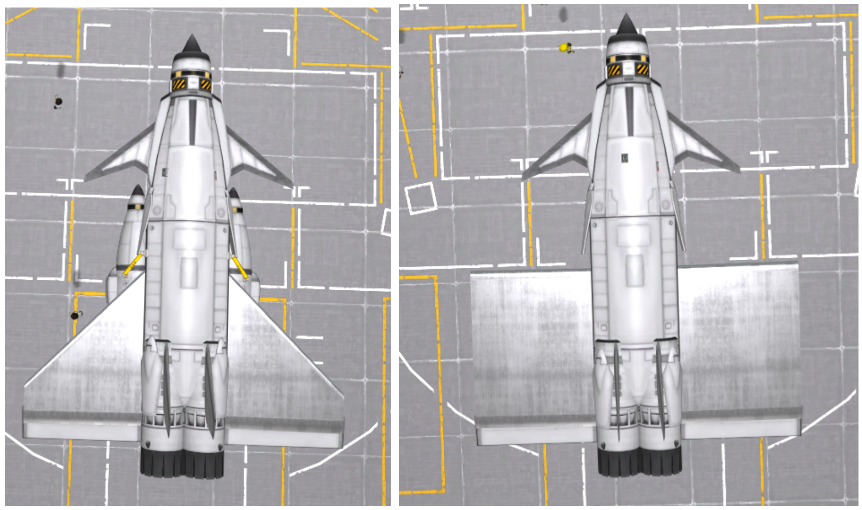

Figure 3: Left, Didact VII 45 degree variation with drop tanks as viewed from above, right Didact VII 90 degree variation without drop tanks as viewed from above.

Once the aircraft was designed, it was flown at 500 meters above sea level, 5 kilometers, 10 kilometers, 15 kilometers, and 20 kilometers from values of Mach 0.8, Mach 1, Mach 1.5, Mach 2, Mach 2.5, and Mach 3. At 25 kilometers above sea level the aircraft was flown from values of Mach 1, Mach 1.5, Mach 2, Mach 2.5, Mach 3, Mach 3.5, and Mach 4. This different Mach range at 25km was due to the difficulty of slowing down to Mach 0.8 at this altitude.

This means that 60 flights were conducted before the data was first analyzed. During the test, the lift-to-drag ratio and the delta-V was recorded using the video recording software Open Broadcast Software. After the first 60 flights, the most efficient angles at each altitude were found. These most efficient angles were flown again at all altitudes to find the get more precise data. 84 flights were conducted in total. A screenshot of KSP with labeled readouts of data is shown below.

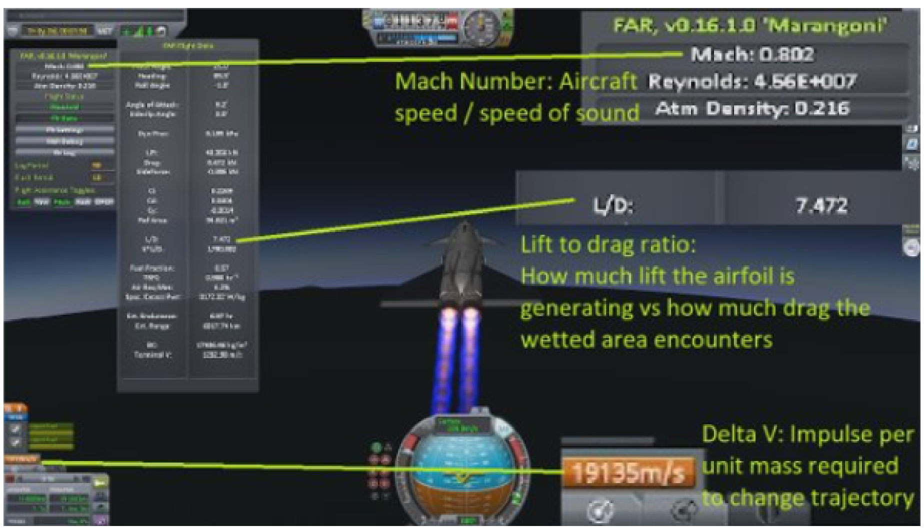

Figure 4: Didact VII in flight preparing to record data, L/D, Mach number, and delta-V shown.

4 Results

After all the flights were recorded, the most efficient delta wing angles at the critical altitudes were found by using a point system. All differently angled flights at an altitude were compared at all critical Mach values. The top three values for both lift-to-drag ratio and delta-V at each critical Mach number were compared. The first place value received 3 points, the second place 2, and the third place 1. Using this point system, the most efficient delta-wing angle was found at each critical altitude. A graphic containing this information is shown below.

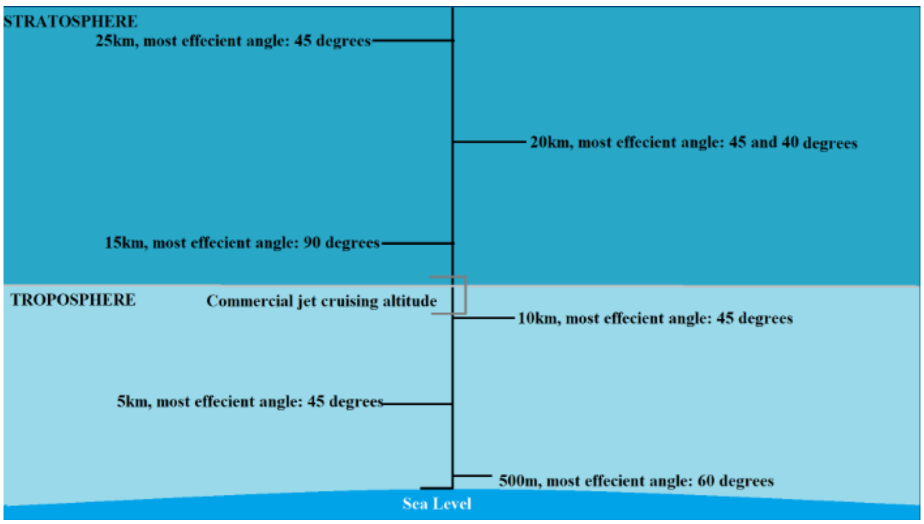

Figure 5: At 500m, the most efficient delta wing angle is 60 degrees, at 5km, the most efficient delta wing angle is 45 degrees, at 10km, the most efficient delta wing angle is 45 degrees, at 15km, the most efficient delta wing angle is 90 degrees, at 20km, the most efficient delta wing angle is tied between 40 and 45 degrees, and at 25km, the most efficient delta wing angle is 45 degrees.

5 Analysis

I then graphed the relationship between Mach angle and delta wing angle delta-V efficiency.

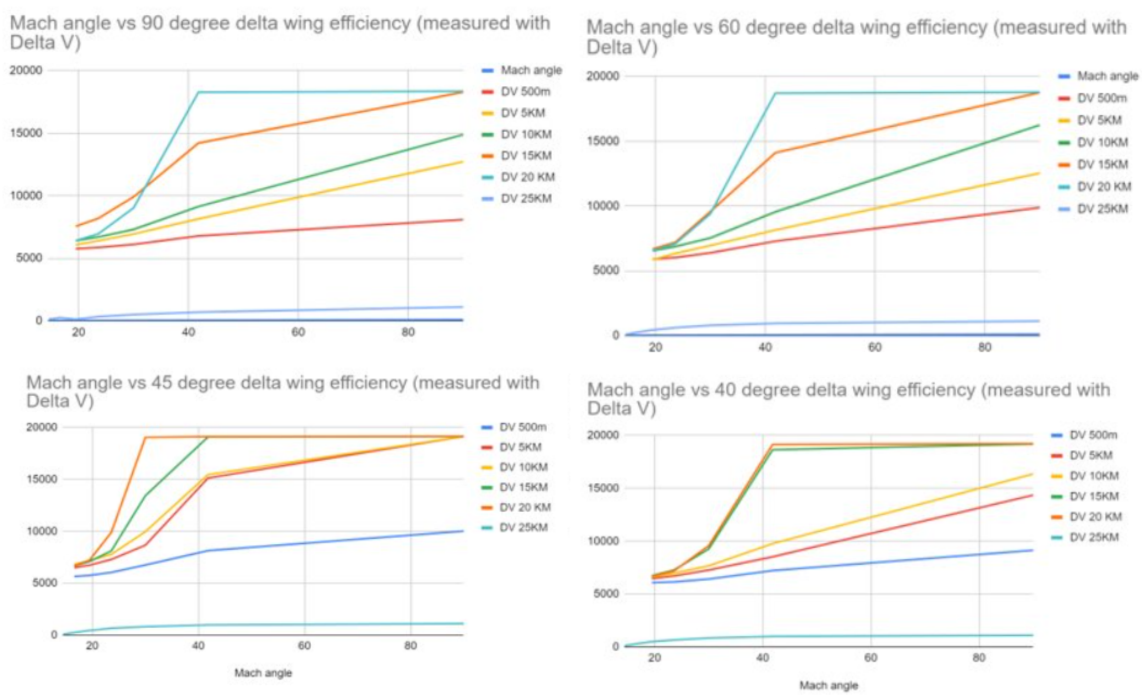

Figure 6: Graphs comparing Mach angle to delta-V efficiency for the 90 degree, 60 degree, 45 degree, 40 degree.

The graphs display the delta-V values for all the most efficient Mach angles. I was expecting there to be a linear relationship between a changing Mach angle and delta wing angle efficiency. I expected that as the Mach Angles become smaller, delta-winged aircraft with larger angles would become less efficient and there would be greater delta-V losses and the lift/drag ratio would decrease.

This relationship did not pan out. 45 degrees was the most efficient which I expected. And as the Mach angle decreased the delta-V efficiency decreased, but the data does not point to any consistent or easy way to model this. Generally, as altitude increases (and therefore air density decreases) the smaller angles become more efficient, but at 15km the 90-degree angle was most efficient, throwing off that at any moderate altitude the 45 would be the most efficient.

In terms of error analysis, I calculated delta-V with the Tsiolkovsky rocket equation.

$$\Delta V = Isp * g * ln[mi/mf] \tag{7}$$

Plugging in the values found on the runway.

$$\Delta V = 4000[N/kg * s] * 9.81[m/s^2] * ln[18.4[tonnes]/11.3[tonnes]] = 19131.38 \tag{8}$$

Comparing this to KSP's value for delta-V, it was found that it was only a 0.36 percent difference. I recalculated this error analysis and got a 0.35 percent difference at 5km, a 0.35 percent difference at 10km, a 0.36 percent difference at 15km, and a 0.36 percent difference at 20km. However, at 25km the percent difference jumped to 92.37 percent. This explains why the 25km lines on the graphs above are so low. The only explanation for this is that KSP has difficulty modeling this altitude, as there is such a drastic difference between the 25km delta-V values and the rest.

6 Bibliography

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