Toy Monte Carlo Simulation of Electron Scattering at Jefferson Lab

Will Craun
Faculty Advisor: Dr. Peter Monaghan

April 20th, 2018

1 Abstract

The objective of the project is to develop a toy simulation of electrons scattering off protons, based upon the experimental set-up in Hall A at Jefferson Lab. The code is developed using ROOT, an open source library, and it generates events based on a model. These events are then used to calculate kinematics quantities which are then compared to real data.

2 Introduction

Nuclear physics by nature is a very complex topic. The entire study is dedicated toward the full understanding and comprehension of the nucleus. One method of study uses electron-proton scattering experiments. These experiments are performed with a beam of electrons impinging upon a static target protons and these scattered particles are detected by electromagnetic spectrometers. This project explores the scattering through a Monte Carlo simulation of the experiment. This means that the code generates randomized data via model to produce results to compare to Jefferson Lab. The simulation uses a random Gaussian event generator as the model for events. The code can produce distribution plots of the modeled kinematics and the acceptances of the spectrometers used in Hall A at Jefferson Lab.

3 Theory

- **Four Vectors**

  The project involves nuclei that move at speeds close to the speed of light ($\sim 3 \times 10^8$ meters per second) and so relativity must be accounted for. Four vectors are a combination of two different types of component information. Time-like is used as the descriptor for the first component and space-like is used to describe the second, third, and fourth components [1].For example, a normal space-time four vector is defined below.

  $$\vec{R} \;=\; < ct, x, y, z > or < ct, \vec{r} >$$

  The four vector's ct term is the time of the event taking place where c is normally set to equal one in order to simplify calculations even further. The x, y, and z terms are representative of the 3 dimensional location of the event. The synthesized use of these 4 components gives the four-vector its computational power.

- **Momentum Transfer Four Vector, Q**

$$Q \;=\; <\omega, P_x, P_y, P_z> or <\omega, \vec{q}>$$

The ω in the time-like component spot represents the energy transfer between the projectile and the static target. The $\vec{q}$ represents the three-momentum transfer between the projectile and the target.Q represents the energy and momentum of a virtual photon which mediates the electromagnetic interaction between the projectile and target. The project does not focus on the interaction its self because the level nuclear physics needed to understand completely is beyond the scope of this project. The virtual photon is simply a tool for the electron to exchange energy and momentum to the proton in order to scatter it.

- **Elastic Electron-Proton Scattering**

Similar to the classical billiards "scattering", the term elastic refers to the fact that energy must be conserved before and after the collision. Elastic also refers to the fact that the target particle will keep their form before and after as well [2]. Figure 1 shows a Feynman diagram of the electron-proton scattering process. Because the type of scattering is an elastic electron-proton experiment, the target is restricted to only be hydrogen ions (free unbound protons). The bold lettering represents a three-vector but the four-vector, "q", seen describing the virtual photon (the squiggle) is our momentum transfer four-vector. The bold **k** and **p** three-vectors represent the electron and proton momenta respectively.

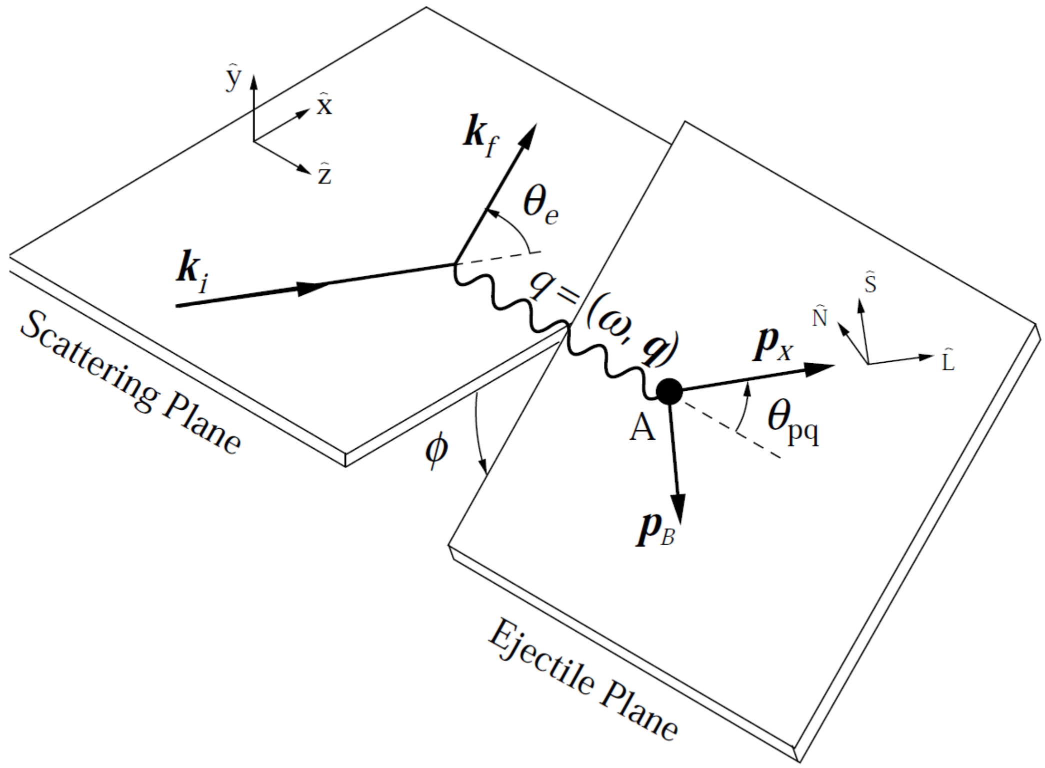

Figure 1: Feynman diagram of electron-proton scattering

The diagram also shows the three dimensional aspect of the experiment. The out-of-plane angle, θ, and the in-plane angle, ϕ, are tools used define kinematics component wise. The angles are an integral part of the vector computations needed for the model.

- **Quasi-Elastic Electron-Proton Scattering and Missing Momentum**

  Quasi-elastic electron-proton scattering refers to a scattering experiment where the electron scatters off a proton in a nucleus that is not hydrogen. With the introduction of a recoiling or "daughter" nucleus, after the interaction the momentum must be conserved becomes an issue [3]. Some of the energy has be used to separate the proton from the nucleus. This is called proton separation energy.

  Along with proton separation energy and the energy lost due to the recoiling daughter nucleus. This leads to the idea of missing energy and momentum which is not observed. Similar to the virtual photon, missing momentum has more meaning than the scope of this project allows to investigate. The term "missing" refers to the fact that the momentum term is not recorded but must be inferred from other observed experimental quantities.

4 Methods

Monte Carlo simulations are commonly used when attempting to model complicated systems with multiple degrees of freedom. By incorporating randomness, these types of simulation can emulate an uncertainty found in all physical experiments. Most models apply predetermined boundary conditions to specify and isolate real world situations that the model attempts to describe. For example, the project used small variances in the scatting angles (θ and ϕ) to reproduce the randomness of real scattering experiments. Some boundary conditions are applied to the particle detector (electromagnetic spectrometer) to control its acceptances.

The Gaussian event generator throws events such that 95 percent of the events fall within both the momentum and angular acceptances. The in-plane angular acceptance is set to $\pm$ 30 milliradians, the out-of-plane angular acceptance is set to $\pm$ 60 milliradians, and momentum acceptance is set to $\pm$ 4.5 percent. These values approximately match the the spectrometer used in Hall A.

To actually simulate the simplified scattering experiment, the project used the open source modular scientific software framework, ROOT [4]. Its great functionality stems from the plethora of classes which make event generation, plot creation, and other processes way simpler. The use of TCanvas and TH1D classes were paramount when constructing the simulation. These classes allowed the code to create histograms that show the distribution of simulated random set of kinematics. The applied randomness/uncertainty is from the TRandom class in ROOT.

The title of "Toy" Monte Carlo comes from the fact that this code does not use an accurate physics model to simulate events. The truth is that not all kinematic quantities are spread in a Gaussian distribution (see Figure 5). Similar to the virtual photon and missing momentum, a more accurate event generator would be out of the scope of this undergraduate project. If the project were continued, the event generation would most likely use the

TGenerator class in ROOT.

5 Data

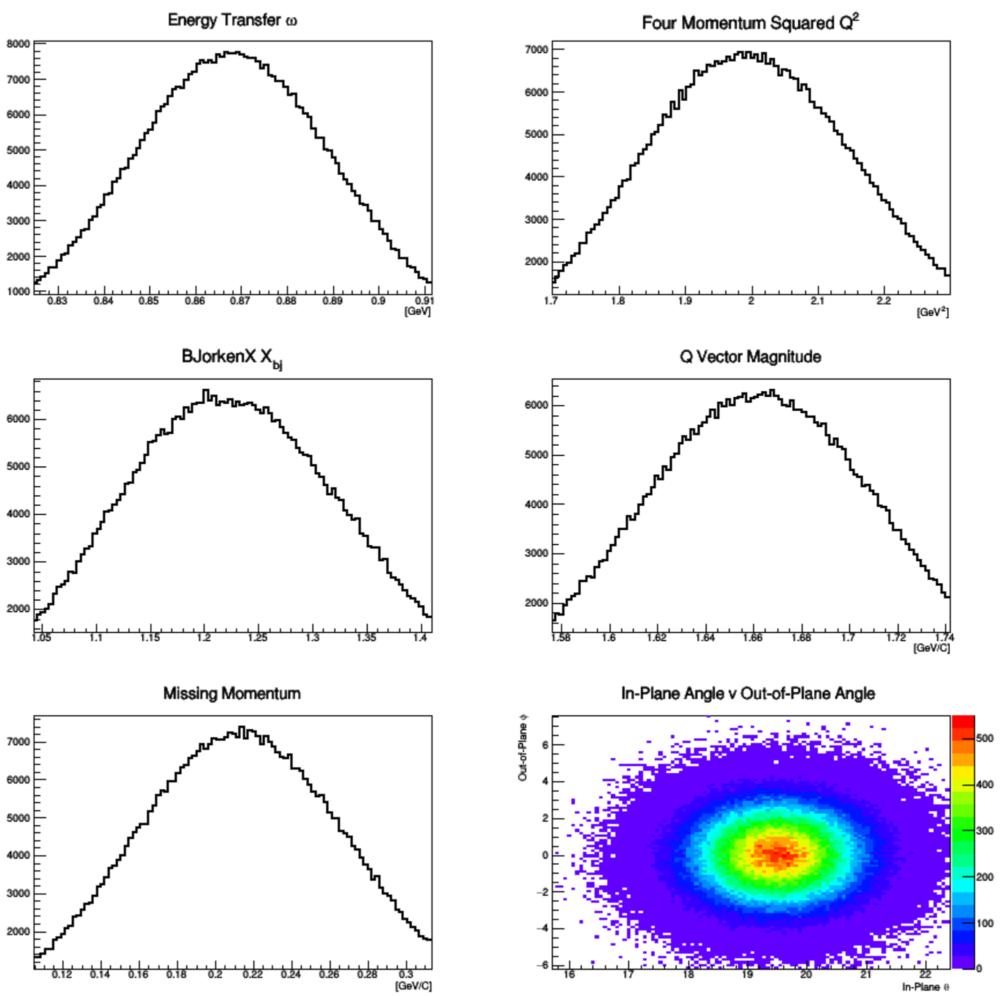

Figure 2 - Set of modeled kinematics where 95 percent of the events thrown fall within the spectrometers acceptances

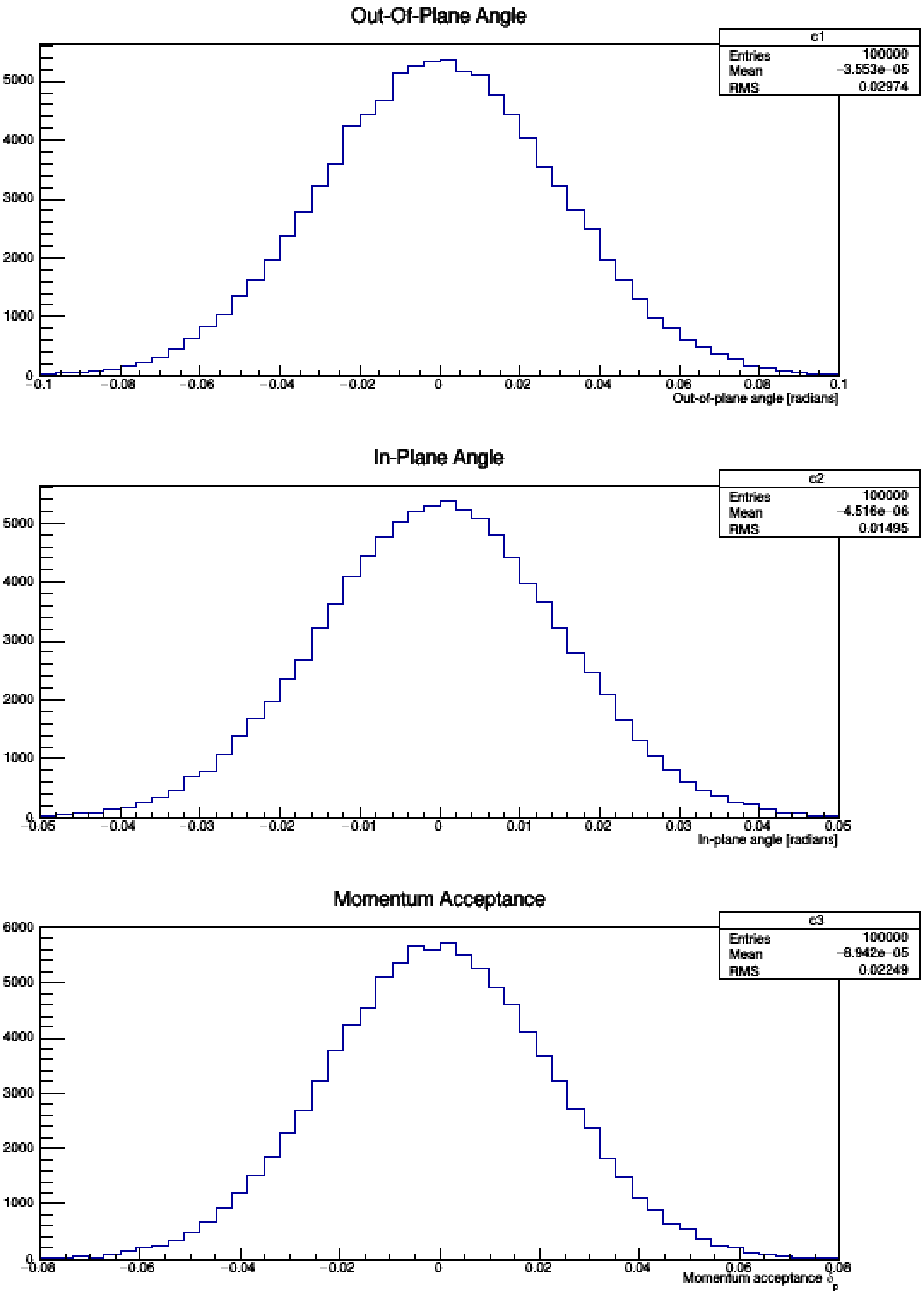

Figure 3 - Acceptance plots produced by the Monte Carlo code

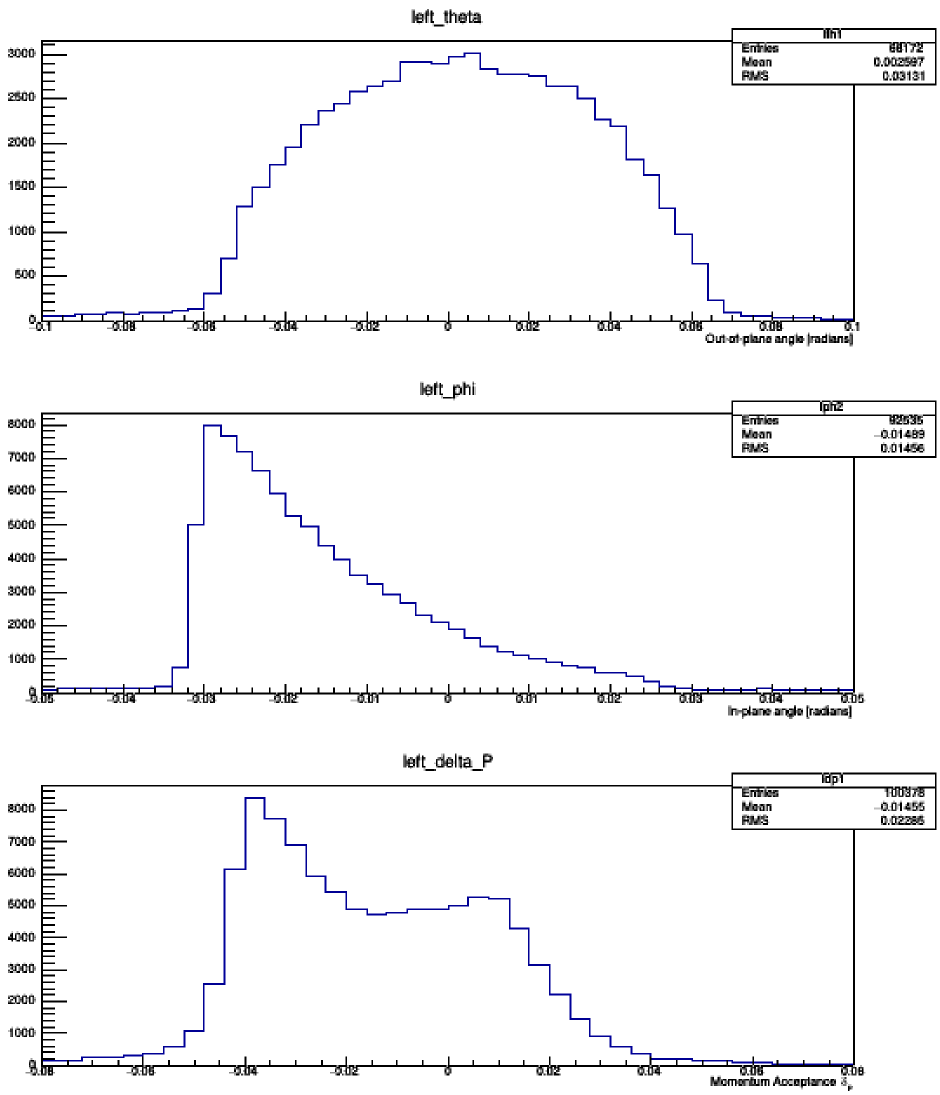

Figure 4 - Acceptance plots taken from Hall A electromagnetic spectrometer. Left arm

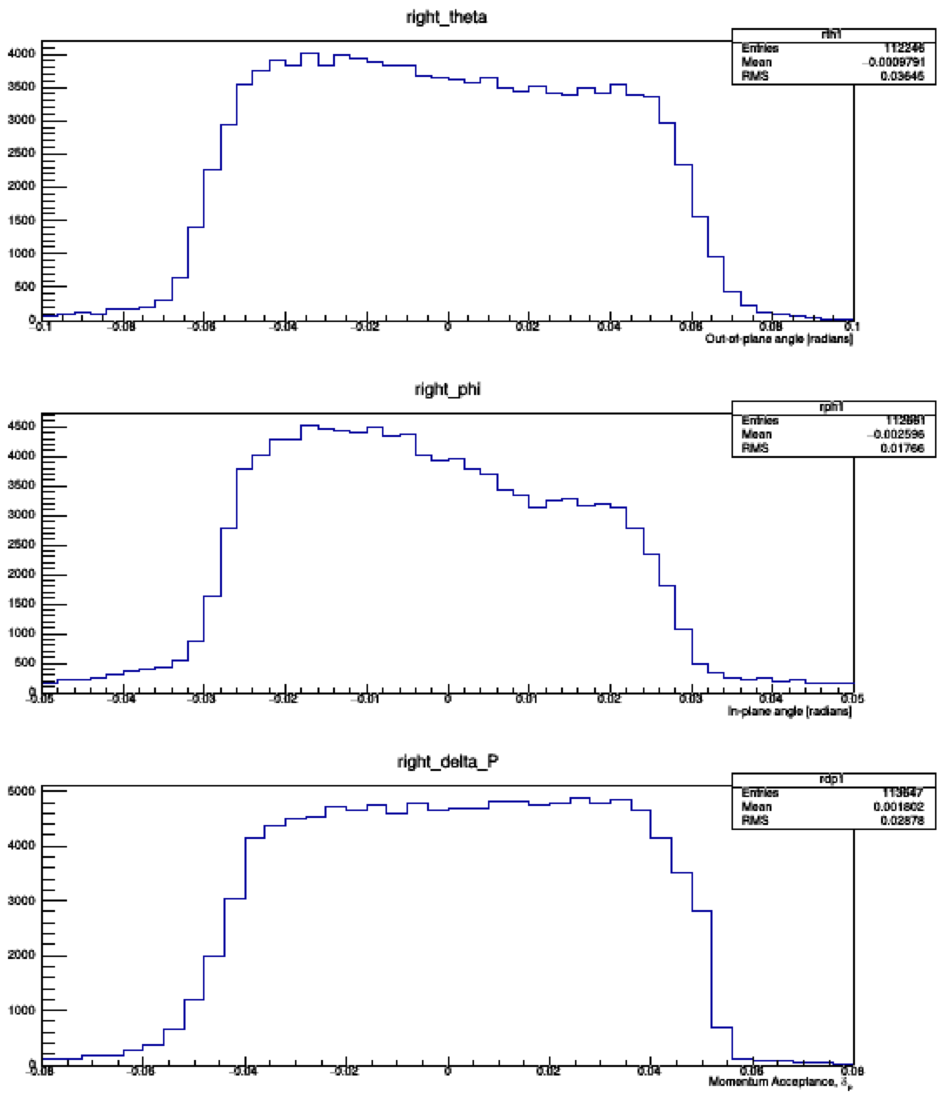

Figure 5 - Acceptance plots taken from Hall A electromagnetic spectrometer. Right arm

6 Discussion and Conclusions

The six plots in Figure 2 show the calculated kinematic values modeled by the Gaussian event generator discussed in the previous section. This is the result of the simplified way to model the probability spread. Plots 1-5 all look very similar due to the inaccuracies of the event generation. However, these plots do reaffirm the fact that only 95 percent of the data was accepted. There are missing sections of the data that account for the missing 5 percent of the thrown events at the edges of the plots. If all 100 percent of the events was plotted within the acceptances, then the ends of the histograms would be at 0.

The last plot shown is a 2 dimensional histogram plot of the in-plane and out-of-plane angles plotted against one another. Red represents more events captured where blue represents less events captured. It demonstrates the fact that the spread of events is densest near the center and shows that the further away from the center lowers the amount of captured events.

Most of the inaccuracies are seen while comparing Figures 3 and 4. The plots in Figure 3 show the simulations acceptances plots and Figure 4 shows the actual acceptances plots taken from the Hall A experiment. These plots represent the acceptances of the left spectrometer arm at Jefferson Lab. The left arm is most often used to detect the scattered electron and the right arm is used to detect the scattered proton. The first plots of both figures show the out-of-plane angle acceptance and are the most similar in distribution. The out-of-plane plot in Figure 3 however has a dome quality that the plot in Figure 2 does not have.

In-plane angle acceptance plots however have very noticeable differences in Figures 3 and 4. The Jefferson Lab plot shows an anti-symmetric distribution that favors angles smaller than the centered angle (the angle at the center of the plot). This effect is described by special relativity [5]. The forward boosted particles create a light-cone effect that tends to lower the in-plane angle value. This is what creates the anti-symmetric distribution shown.

Similar to the in-plane plot from Figure 4, the momentum acceptance is anti-symmetric but has a more spread out distribution toward positive values. This is because scattering experiments are messy. The incident particles can bump or get disturbed by other particles, scatter off multiple protons, or scatter off an unintentional target (eg. neutron). Any one of these possibilities could be the reason behind the distribution but the experimenters have no way of knowing. Looking now at Figure 5 (right arm acceptances), these distributions show the effect with even more severity. As stated before, the right arm detects scattered protons which have more opportunities to interact with its environment. Incident protons in quasi-elastic scattering are first "hit" while still attached to the target atom. The proton separation energy already affects the scattered particle but the proton could easily collide/interact with other nucleons while leaving the daughter nucleus.

7 Glossary

- Acceptance - The range of values observed and recorded by the spectrometer.
- Elastic Scattering- a collision of beam particles and nucleon targets where the kinetic energy from the incident particles is conserved.
- Quasi-Elastic Scattering- a collision of beam particles and a static target where a single nucleon is scattered from the recoiling nucleus.
- Four-momentum Transfer Vector- Q $[\omega, \vec{q}]$, a four-vector with an energy transfer time-like component and a momentum transfer space-like component.
- Energy Transfer- ω, the amount of energy exchanged during the collision between the beam particle and the target.
- Momentum Transfer- $\vec{q}$, the amount and direction of momentum being transferred to the target particle during the collision.
- Bjorken-x- X_{bj}, a kinematic quantity dependent on Q^2 and ω used to describe the differences in scattering experiments (eg. quasi-elastic electron-proton scattering the value is 1)
- Virtual Photon- a particle that behaves like a normal particle but cannot be detected during the experiment. It is commonly used to indicate the interaction between particles.
- Missing Energy- the energy that is not recorded by the detectors.

- Missing Momentum- the momentum that is not recorded by the detectors.
- Excitation Energy- it is a discrete amount of energy added to a system that changes the state of the system from ground to an excited state. Systems refers to nuclei, atoms, and other molecules.
- Proton Separation Energy- the amount of energy required to remove a proton from a nucleus.

8 Equations

*note M_p represents for mass of a proton in $[\frac{GeV}{C}]$, E represents the beam energy in [GeV], and E' represents the incident energy of the scattered electron in [GeV]

$$Q^2 \ = \ -q^2 \ = \ 4E_eE'_e sin(\frac{\theta}{2})^2$$

$$\omega = E - E'$$

$$X_{bj} = \frac{Q^2}{2M_p\omega}$$

References

[1] David J. Morin. *Special Relativity for the Enthusiastic Beginner.* CreateSpace, 2017.

[2] Mott-Massey. *The Theory of Atomic Collisions.* Clarendon, 1949.

[3] Kenneth S. Krane. *Introductory Nuclear Physics.*

[4] Root user guide, https://root.cern.ch/root-user-guides-and-manuals.

[5] N.M.J Woodhouse. *Special Relativity.* Springer-Verlag London, 2003.