

# Designing, Constructing and Testing a Stirling Engine

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## 1 Abstract

This project was undertaken with the goal of designing a heat engine, which could be built and analyzed, that would utilize the Stirling cycle. The engine was meant to use an input of heat to output work, which could be measured and in turn used to determine specifications about the performance of the engine. Some of these specifications include the efficiency, maximum revolutions per minute, and the maximum torque output by the engine. In addition to engine specifications, relationships between the thermodynamic state variables and the measured output quantities of the engine could be observed and analyzed. With these goals in mind, over the course of six months, the Stirling engine was designed, modeled, built, tested, and analyzed. The majority of the six month time frame was used for the planning and modelling of potential engines, with the final two months spent building the selected design and testing its functionality.

## 2 Introduction

A major reason for the selection of this project was an interest in heat engines, and the utilization of thermodynamic processes to produce work. The Stirling engine specifically, is an example of a very quiet, fairly simple engine, with the prospect for a very high efficiency and a very clean operation. Unlike engines utilizing the Diesel or Otto cycles, such as the petroleum or diesel combustion engines, the Stirling engine does not require the combustion of any fuel, but rather the transfer of a working fluid. This allows functioning Stirling engines to be built with lower tolerances and lower material costs. I elected to build an engine utilizing a combination of previously successful Stirling designs. Stirling engines come in three main forms, with each utilizing a different orientation of pistons to force the engine to cycle continuously and produce a constant work.

For the engine built in the course of this project, an electric stove top burner was used as a heat source, and a Prony break was used to measure the power output of the engine. During data collection, a variety of tests were run with different sets of independent variables, to gather data relevant to the relations between the varying of thermodynamic state variables and changes in the output of the engine.

### 3 Theory

#### 3.1 The Stirling Cycle

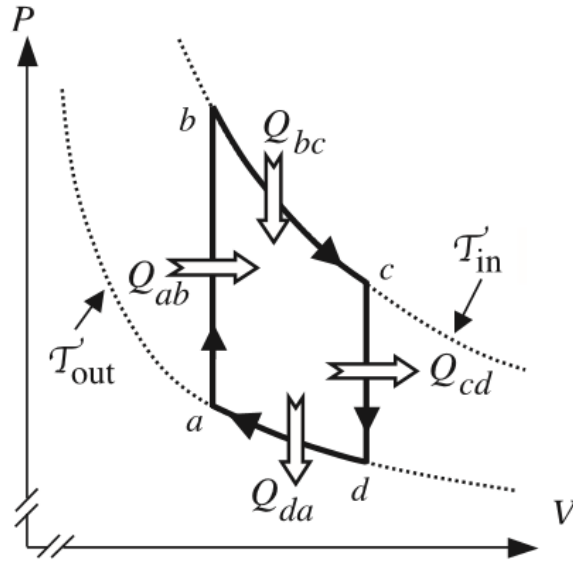


Figure 1: Stirling Cycle PV Diagram

Much of the mathematics of the project is associated with the thermodynamic processes in the Stirling cycle. The Stirling cycle is comprised of isochoric and isothermal expansion and compression. In an isochoric process, the volume of the system is kept constant. This means that since work is given by the formula,

$$W = \int P dV$$

and the change in volume is 0, no work is produced. From this, we can safely assume that all of the work produced by the engine comes from the isothermal expansion and compression. Therefore the work that the engine produces is given by,

$$W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

where  $n$  is the amount of working fluid in moles,  $R$  is the ideal gas constant, and  $T$  is the temperature. The term inside of the natural log is the compression ratio. This is the ratio of the maximum volume of the engine to the minimum. This simplifies the calculation of the engine efficiency, which is given by the equation,

$$\eta = \frac{W}{Q}$$

where  $Q$  is equal to the heat input to the engine to allow for continuous cycling. The power that the engine produces can be found using the equations

$$P = \omega * \tau$$

$$\tau = F * R$$

$$\omega = RPM * \left(\frac{2\pi \text{ radians}}{60 \text{ seconds}}\right)$$

where  $\omega$  is the angular velocity of the engine's flywheel. This can be found by multiplying the revolutions per minute of the engine by  $2\pi$  radians, signifying a full cycle, and  $60 \text{ s}^{-1}$  to get the final units of radians per second. The torque,  $\tau$ , can be found by multiplying the tangential force of the spinning flywheel by the radius of the wheel. The heat input to the engine can be found using the equation,

$$Q = mc\Delta T$$

where  $m$  is the mass of the working fluid, in this case air,  $c$  is the isovolumetric heat capacity of air, and  $\Delta T$  is the change in temperature of the system. Since the full output of the heat source will not be evenly received by the engine, this equation will be used to approximate the total heat input.

### 3.2 The Importance of the Regenerator

The Stirling cycle is greatly complicated by the introduction and expulsion of heat at two separate points in the cycle. To show this I have calculated the efficiency of an engine that runs purely on the Stirling cycle to be,

$$\eta = \frac{T_h - T_c}{T_h + \frac{C_v(T_h - T_c)}{nR \ln(V_f/V_i)}}$$

By introducing the regenerator, we can better conserve the thermal energy in the engine. The regenerator is made of a material with a high thermal conductivity. This way, heat introduction and removal is mitigated in the isochoric processes. This allows us to change the efficiency of the engine to,

$$\eta = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$$

This efficiency is significant because it is equal to the Carnot Efficiency. This makes for a much more efficient engine. This also helps with the continued operation of the engine. As the temperature of the regenerator increases, it is able to supply the engine with more and more thermal energy. This allows the engine to continue operation even when no heat is being introduced, until the regenerator cools down.

## 4 Methods

### 4.1 Stirling Engine Types

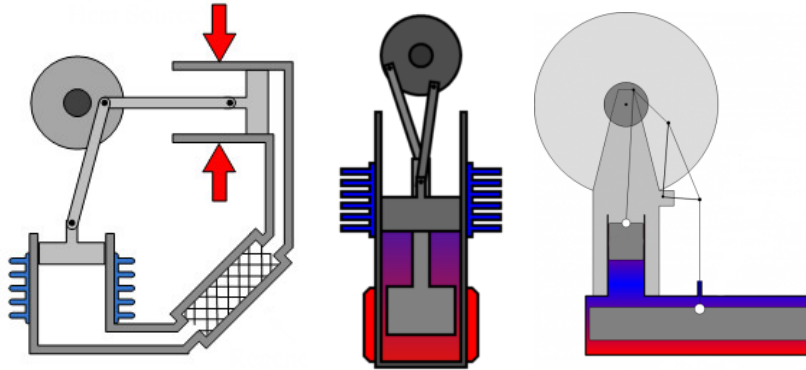


Figure 2: Alpha (left), Beta (middle), and Gamma (right) Type Engines

In order to facilitate the Stirling cycle, Stirling engines must separate two areas to have a hot and a cold reservoir. The piston attached to the hot reservoir is referred to as the power piston, while the piston attached to the cold

reservoir is named the displacer piston. In order to accomplish this, Stirling engines are generally designed to fit into one of three forms, alpha, beta, and gamma-type Stirling engines as shown in figure 2. The alpha-type engines have two cylinders, opposed by a  $45^\circ$  angle, separated by the engine's regenerator. The beta-type opts to use a single cylinder with two pistons, one of which is augmented with an integrated regenerator. Finally, the gamma-type uses the integrated regenerator of the beta type, but focuses the cold reservoir down to a smaller radius cylinder.

## 4.2 Construction

Two engines were designed for this project, one pure alpha-type Stirling engine, and one that combined some of the characteristics of both the alpha and beta-types. The alpha-type engine was constructed from pine, and used two Pyrex test tubes for the engine cylinders. Being an alpha-type engine, The two test tubes were positioned at a  $45^\circ$  offset from each other. This first engine never reached the point in the fabrication process where it could cycle continuously. This can be attributed to a lack of momentum in the flywheel and an inadequate bore-to-stroke ratio. The problem of the momentum could be fixed by simply fabricating a stronger crankshaft and attaching a more massive flywheel. The bore-to-stroke ratio would be a much more difficult problem to fix. The pistons of the first engine had a diameter of 2.5 centimeters, with a stroke of 1 centimeter. This gave the engine a bore-to-stroke ratio of 2.5, producing inadequate torque to move the engine through its cycle continuously.

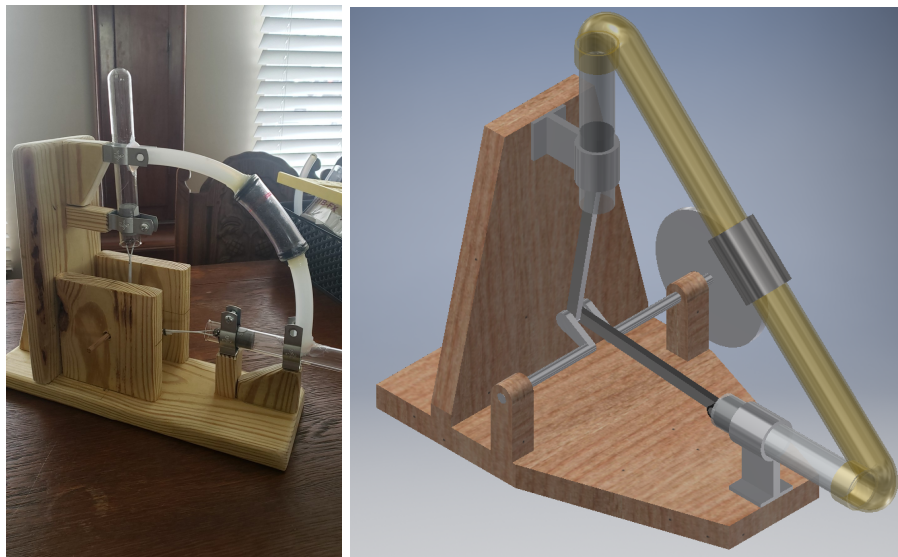


Figure 3: First Engine Actual (left), and Model (right)

The second engine used the integrated regenerator of the beta-type engine, with the  $45^\circ$  piston offset of the alpha-type engine. The benefit of having a

regenerator integrated into the power piston is that there is a much shorter distance for the input heat to travel before reaching the regenerator. This decreases the chance of heat escaping through the walls of the engine. By separating the hot and cold reservoirs, you increase the distance between the hottest and coldest part of the engine. This makes it much easier to keep the reservoirs at two distinct temperatures, instead of melding into one. The second engine also has a much larger bore power piston than the first. The diameter of the power piston is 7 cm on the second engine, giving it a bore-to-stroke ratio of 7. This produces a great deal more torque with the same heat input. It also uses a diaphragm for the displacer piston to allow the piston to have a perfect seal and keep the engine at a constant 1 atmosphere of pressure inside.

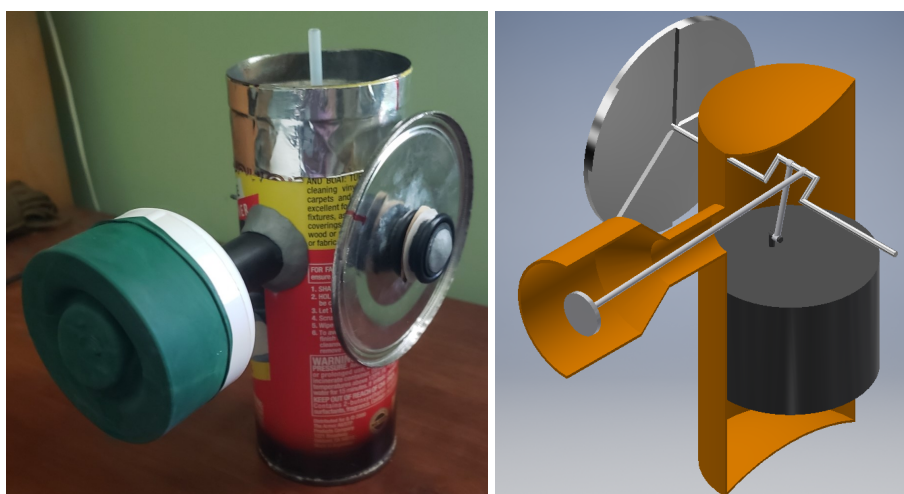


Figure 4: Second Engine Actual (left), and Model (right)

### 4.3 Data Collection

In order to measure the power output of the engine, a Prony brake was built. A Prony brake is a device that measures the force needed to slow the engine to a certain number of RPMs. The brake slows the engine by applying force above and below the crankshaft axle. The increased friction slows down the engine, and a bar protruding off of the brake attaches to a digital scale to measure the change in force. Data can be recorded for a number of different engine speeds and forces to get power and torque curves for the engine. The digital scale used was capable of measuring mass to the nearest .5 grams. In order to measure the speed of the engine for the various tests, Samsung's "Super Slow-mo" functionality for the Samsung Galaxy S9 was used to slow a video of the engine's operation. While being viewed in "Super Slo-mo," the video is slowed down to 1/32 the speed of the original video. This allows the revolutions in the video to be counted and multiplied by 32 to get the actual RPMs of the engine.

The first test performed used the Prony brake and a constant temperature heat source. The speed of the engine and the mass measured on the scale were

both recorded for the first experiment. A secondary test was performed without the use of the Prony brake. The engine was allowed to spin unhindered, but the heat input was varied. the engine speed was again recorded using "Super Slo-mo" functionality. Each speed was recorded and paired with a temperature measurement for the heat source of the engine. For both the second and the first test, a 3200 watt, 10 inch electric stove top burner was used as a heat source.



Figure 5: Prony Brake

Data was processed using python code that took in a list of force, RPM pairs and returned the associated torques and powers. The data was also plotted using matplotlib to show what kind of relationship if any existed between the engine speed, the torque, and the power at a constant temperature. For the second test, the program plotted the speed of the engine against the heat input to the engine to attempt to see a relationship between the two.

## 5 Data

The data taken for the first test used a constant heat input from the 3200 watt, 10 inch electric burner. Temperature of the heat source was taken before and after each data point was collected to ensure that the temperature never deviated from the average of  $353.7^{\circ}$  Celsius by more than  $\pm 2^{\circ}$  Celsius. The same force and angular velocity pairs were used to plot power and torque. This

serves to show a comparison between the torque that the Stirling engine was able to produce and the power output of the engine.

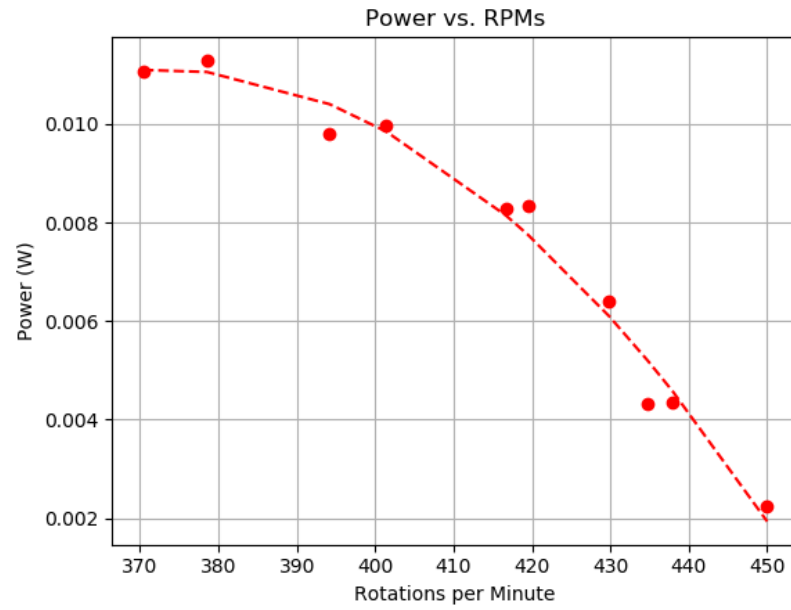


Figure 6: Power (W) vs. Engine Speed (RPM)

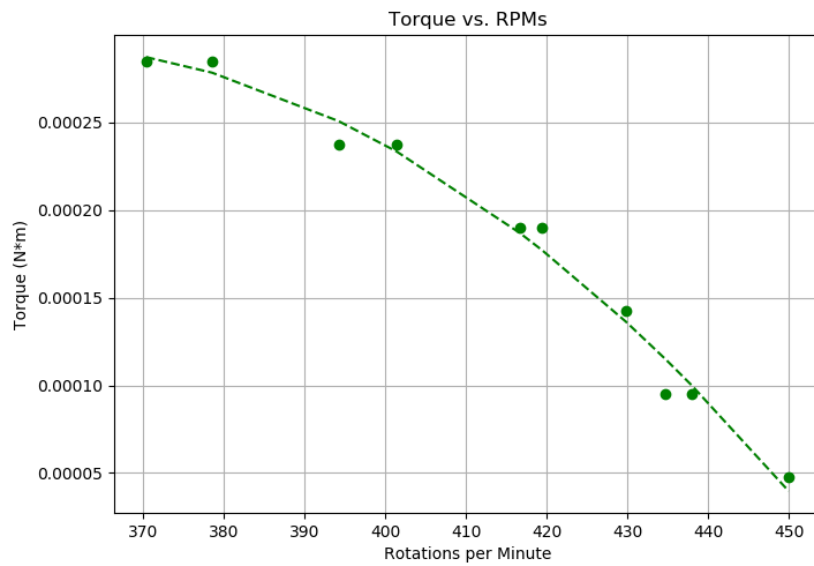


Figure 7: Torque (N\*m) vs. Engine Speed (RPM)



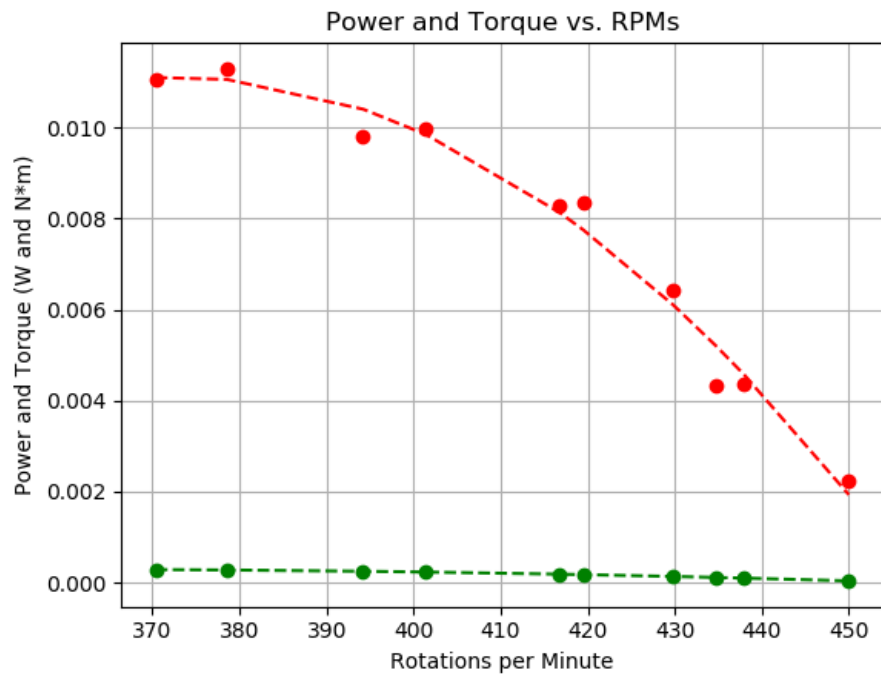


Figure 8: Power (red) and Torque (green) Curves vs. Engine Speed (RPM)

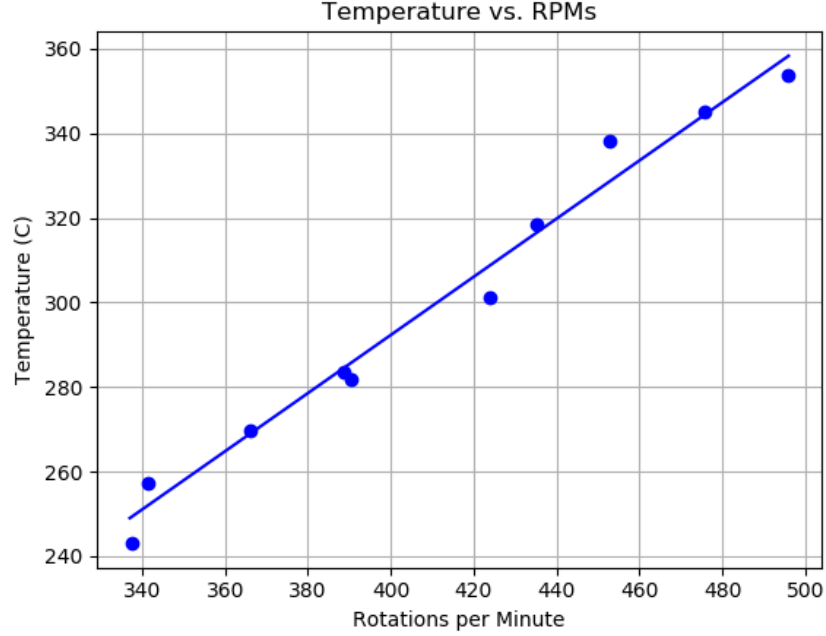


Figure 9: Temperature (C) vs. Engine Speed (RPM)

The second test was performed by slowly increasing the temperature of the electric stove top burner. Each time a video was taken to record the engine speed, the temperature was taken both before and after, and the average was used in the data set. Since the temperature was increased slowly, none of the before/after temperatures deviated from each other by more than  $\pm 3^\circ$  Celsius. Temperature measurements for all tests were performed using the same IR thermometer.

## 6 Discussion and Conclusions

In regards to the first test, at a constant temperature, data for the power and torque can both be described by a quadratic function. Both show an inverse relationship between the number of revolutions per minute and the work output of the engine. The maximum force that the Prony brake was able to apply before the engine stopped was 3 grams. When this was compared to the lowest engine speed measured, 370.37 RPMs, the python script yielded a power of 0.0113 watts and a torque of  $2.85 \times 10^{-4}$ . A maximum power of 0.0113 J/s is less than expected for the Stirling engine constructed in this project. This can be attributed in large part to the much higher bore-to-stroke ratio of the engine. With a high bore to stroke ratio, an engine is capable of producing more torque than the same engine would with a lower bore-to-stroke ratio. Stroke is defined

as the length of the crank on the crankshaft. As the stroke increases, the engine is capable of producing more work during one complete cycle, however, more force must be applied on the cylinders to push the engine through a revolution. This results in an engine with a slower angular velocity. This means that if the engine is capable of continuous operation with a low bore-to-stroke ratio it will be more efficient in its operation. Unfortunately, in order for the engine built in this project to achieve continuous revolutions with the heat source used, the engine needed a shorter stroke and a faster engine speed.

Figure 3 was included to help to illustrate that the curves would eventually reach a point where the speed was maximized and the torque and power near zero. The maximum engine speed was determined during testing to be 495.87 revolutions per minute. If the digital scale used to measure the data was precise enough, then the curves would be able to show that they each approach this value.

Because of the area of the heat receiver on the engine, the engine did not receive all 3200 watts from the burner. Using the equation for calculating heat in the system, The total heat input to the system was calculated to be 0.893 joules every second. This gives a maximum efficiency of 0.0126 for the experimentally determined efficiency. This was found by comparing the work output by the engine to the heat input. Using the ideal efficiency formula, which only takes into account the maximum and minimum temperatures of the engine, the efficiency was calculated to be 0.54. This is a great deal better than the efficiency determined by using the work and heat of the engine. This suggests that the engine built for the project, while still a functioning Stirling engine, was a great deal different from the perfect ideal Stirling engine. Only by using the ideal Stirling engine would the efficiency calculated by the two equations be the same.

For the second test, in which the heat input was varied, the relation between the engine speed and the temperature proved to be directly proportional. This makes sense, as an increase in heat would give the engine more fuel to work with. As heat increases,  $\Delta T$  grows, and all other factors in the isothermal equation for the determination of work stay the same. The linearity of the plot is described in the same way, because there is a 1:1 relationship between work and temperature.

Overall I see this project as a success, despite the differences in efficiency and the lower than expected power output of the engine. Moving forward, I believe that it would be interesting for any successors to construct a similar engine out of higher quality materials, with higher manufacturing tolerances and compare their results to those determined in this project. The use of a material with a higher heat capacity than the housing of the engine could improve the overall efficiency by eliminating a portion of the escaping heat. A change of the working fluid from air at atmospheric pressure to a pressurized monatomic gas such as helium could also improve the efficiency of the engine. A monatomic gas would have a lower specific heat capacity, and thereby allow heat to flow through the system more effectively.

## 7 Appendix



Appendix A: Second Engine Side View



Appendix B: Second Engine with Prony Brake

## 8 Bibliography

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