Increasing the Accuracy and Effectiveness of the Millikan Experiment

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Abstract

The goal of this project was to increase data accuracy of the Millikan Experiment that was formerly a part of the PHYS 202 lab. The lab was pulled from the curriculum due to its inaccurate data that had a wide range of final calculated values of the charge of an electron that were too far off from the theoretical value. Through this project, I was able to gain a much better understanding of why the data gathered was so inaccurate, and I have found ways to make the experiment more effective, as well as easier to conduct.

1 Introduction

This project was designed to answer two questions. The first question to be answered was whether or not the data gathered from the Millikan Experiment could have increased accuracy. Originally, the values for the calculated charge of an electron differed greatly from that of the theoretical value. They also had a large distribution, making the data difficult to understand. The second question to be answered was whether or not the experiment could be improved to reintroduced it into the academic laboratory setting.

Before the project was conducted, it was known that the latex droplets falling through the chamber did not have a uniform radius between them. This was a primary focus of the project, because it would help to answer both questions. This brought the need for a new data collecting method to be introduced, as well as the potential need for new or different equipment.

2 Theory

2.1 Calculating the Fundamental Charge of an Electron

The main equation used in the Millikan Experiment is that to determine the charge of the latex spheres falling through the chamber. That equation is given as follows:

$$q = \frac{\rho(\frac{4}{3}\pi r^3)gD}{V} \tag{1}$$

where ρ is given to be the density of the spheres, r is the radius of the droplet, g is the acceleration due to gravity, D is the distance between the capacitor plates, and V is the measured stopping voltage.

2.2 Calculating the Diameter of the Droplets

The radius of the latex spheres falling through the chamber is calculated as follows:

$$r = \sqrt{\frac{9\eta v}{2\rho g}} \tag{2}$$

where η is the viscosity of the air, v is the velocity of the falling spheres, ρ is the density of the latex, and g is the acceleration due to gravity. Coupling these two equations in the experiment customized each equation to match each sphere better—by giving an accurate radius to each droplet—giving less uncertainty and more accuracy in the calculations.

2.3 Error Analysis

In a project conducted by Yingzi Li *et. al.*, an error analysis was performed to determine whether or not the best method of collecting data was by measuring the stopping voltages as well as measuring the fall rates. The error analysis is given as follows:

$$\Delta Q = \sqrt{(\left|\frac{\delta Q}{\delta t_g}\right| \Delta t_g)^2 + (\left|\frac{\delta Q}{\delta t_e}\right| \Delta t_e)^2 + (\left|\frac{\delta Q}{\delta U}\right| \Delta U)^2} \tag{3}$$

$$\left|\frac{\delta Q}{\delta t_g}\right| = \frac{K'[t_e(6 + 9A't_g^{\frac{1}{2}}) + (2 + 5A't_g^{\frac{1}{2}})t_g]}{4Ut_e((1 + A't_g^{\frac{1}{2}})t_g)^{\frac{5}{2}}} \tag{4}$$

$$\left|\frac{\delta Q}{\delta t_e}\right| = \frac{K'}{U^2(1 + A't_g^{\frac{1}{2}})^{\frac{3}{2}} t_g^{\frac{1}{2}}} \tag{5}$$

$$\left|\frac{\delta Q}{\delta U}\right| = \frac{K'}{U^2(1 + A't_g^{\frac{1}{2}})t_g^{\frac{1}{2}}} \tag{6}$$

In the above equations, t_g and t_e are rise and fall times of the particles respectively, U is the stopping voltage, and Q is the calculated charge. A is Eq. 2 and $K = A^2$. The above equations were then simplified as follows:

$$\left|\frac{\delta Q}{\delta t_g}\right| = \frac{3K'}{4U} \frac{2 + 3A't_g^{\frac{1}{2}}}{(t_g + A't_g^{\frac{3}{2}})^{\frac{5}{2}}} \tag{7}$$

$$\left|\frac{\delta Q}{\delta t_e}\right| = 0 \tag{8}$$

$$\left|\frac{\delta Q}{\delta U}\right| = \frac{K'}{U^2} \left[\frac{1}{t_g(1 + A't_g^{\frac{1}{2}})}\right]^{\frac{3}{2}} \tag{9}$$

Upon completing this analysis, it was found that selecting a drop with a large fall rate and a large stopping voltage would yield computed charges within fewer multiples of the elementary charge of an electron this would make sense as it would ensure that the particles have acquired enough charge, don't experience too many effects of gravity, or experience any clumping.

3 Methods

3.1 Initial Data

Data was obtained by running the experiment normally. This gave a baseline to see if the experiment was getting more accurate through each phase.

3.2 Gathering Fall Rates

The first part of the experiment consisted of testing the sizes of the particles being measured. This required letting droplets fall through the chamber without any applied voltage. By taking a video and carefully timing the fall rates of the particles, a more accurate distribution of diameters were determined, given in Figure 3.

3.3 Introducing New Data Collecting Methods

After gathering a new set of particle diameters, it was determined that introducing a new average diameter would not be sufficient. Because the particles varied too much in size, with a standard deviation of $\sigma = 2.577$, each particle's fall rate needed to be measured individually along with their stopping voltages to get a more accurate charge calculation. With this, the "Measure & Measure" method was introduced into the experiment—a method in which fall rates and stopping voltages would be collected in the experiment. By doing this, each particle would have more 'custom' formula for charge calculation.

3.4 Introducing New Atomizers

One method proposed in this project was using a different type of atomizer. The reason behind this was the hope that a different material—or stem length—of the atomizer would either decrease the random amounts of charge accumulated on the drops, or decrease the range of droplet sizes. I experimented with the two different atomizers—a travel spray bottle and a nasal sprayer. Neither of these sprayers were able to consistently produce a quality cloud of droplets inside the chamber, so they were not able to be used further. The atomizers originally used for this experiment are no longer functional, but I was able to get one from Ron Hanchin that was part of a large batch that he recently ordered. These atomizers work perfectly fine for the experiment, however, their only downside is that if their bulb isn't squeezed hard enough, the liquid comes out in large clumps.

4 Data

The below graphs represent the computed charges after each phase of the project. The histograms show the calculated charges, with how many times they occur in their given range. The last graph, Figure 6, shows a scatter plot of the computed charges with a line representing the elementary charge of an electron. The value of this graph is that it helps to show the points getting closer to the necessary value with each phase of the project. These graphs are given beginning on the next page.

4.1 Initial Trial Data

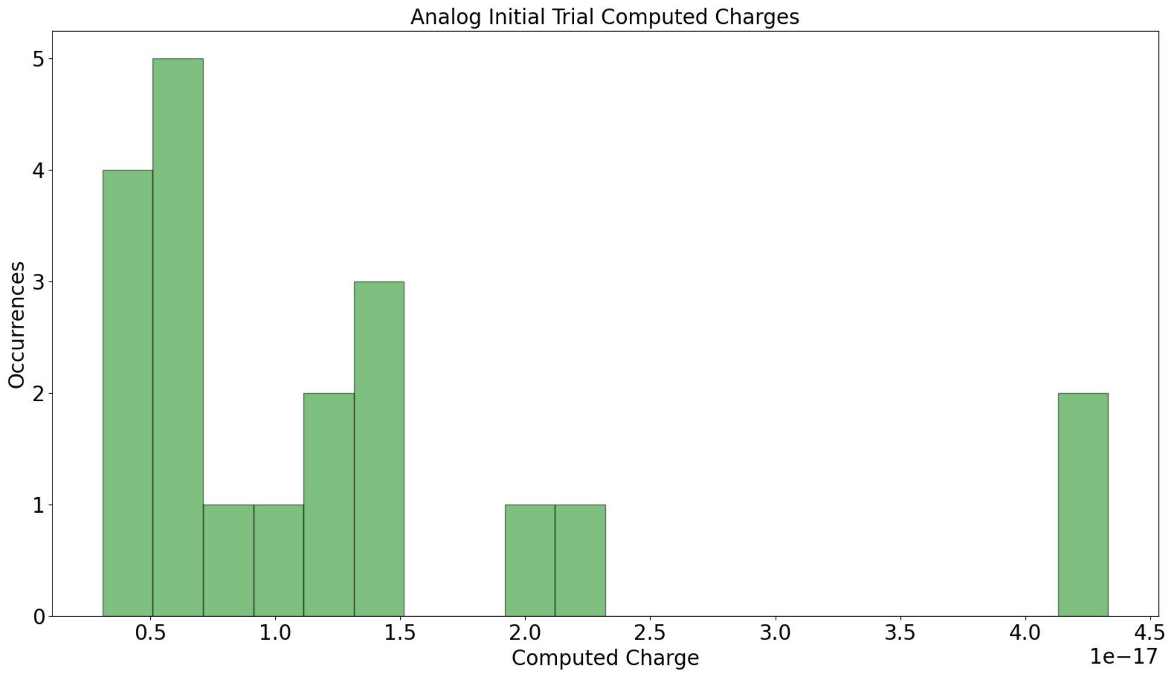

Figure 1: Analog Initial Trial Data

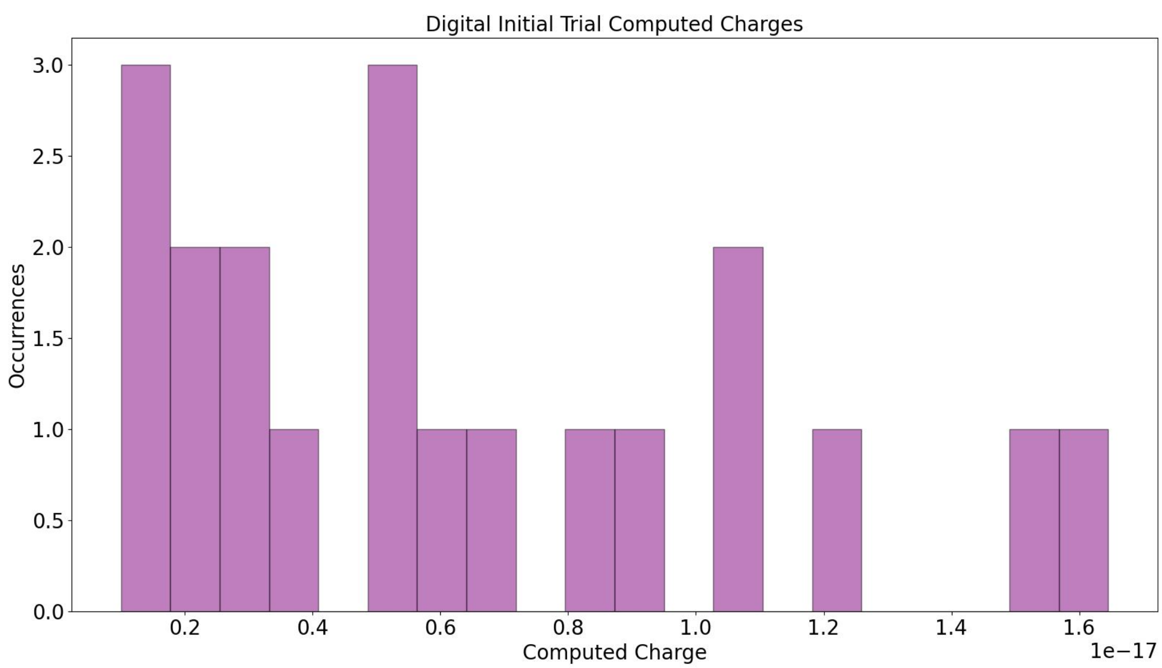

Figure 2: Digital Initial Trial Data

4.2 Fall Rate Data

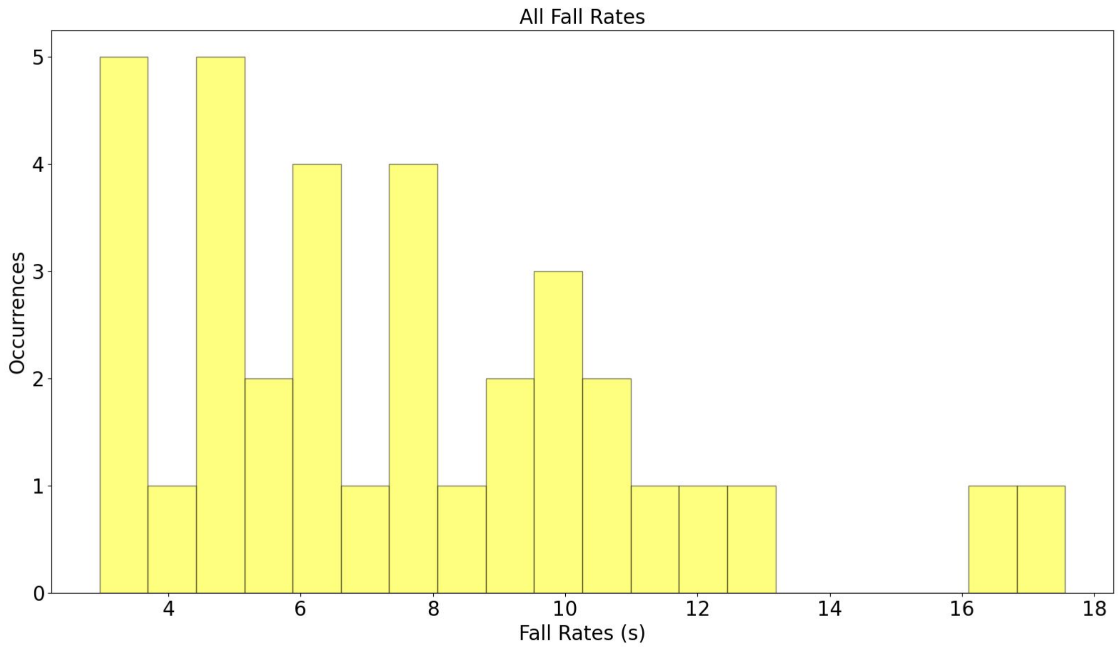

Figure 3: Fall Rate Data

4.3 New Mean Radius Data

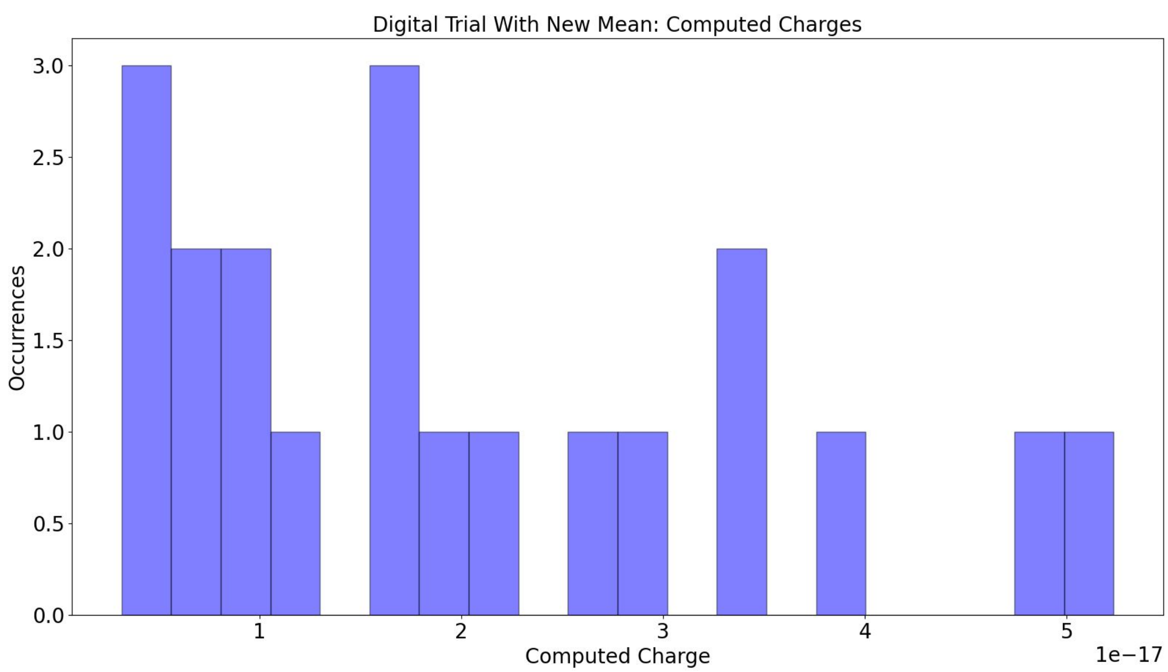

Figure 4: New Mean Radius Trial Data

4.4 Measure & Measure Data

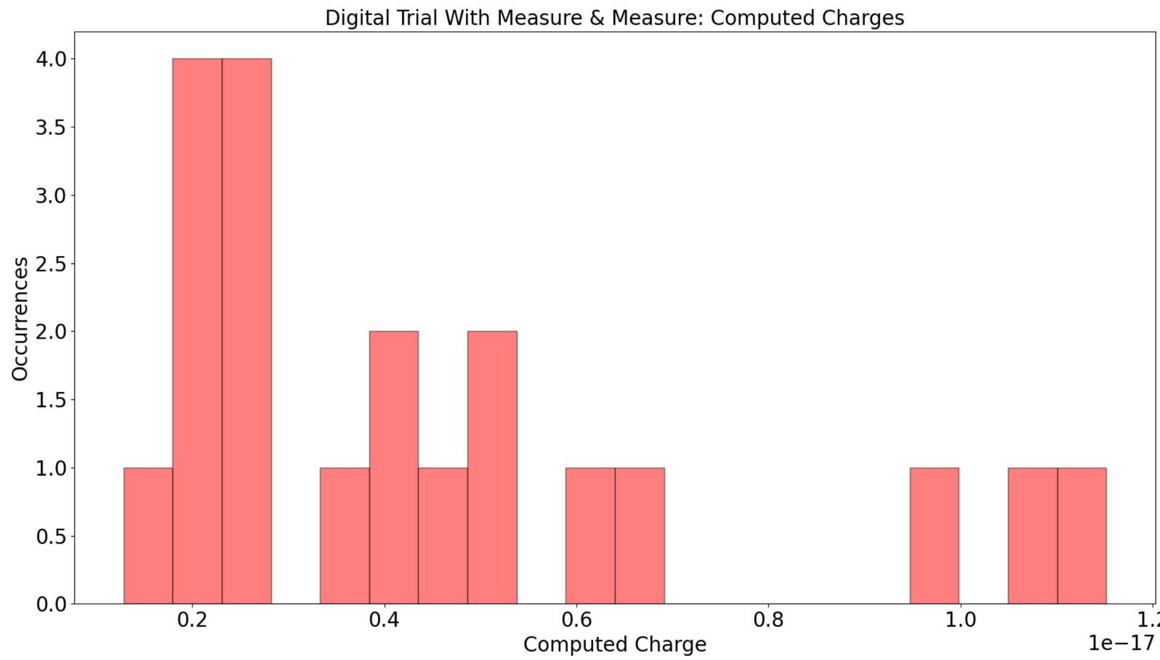

Figure 5: Measure Measure Data

4.5 All Trials

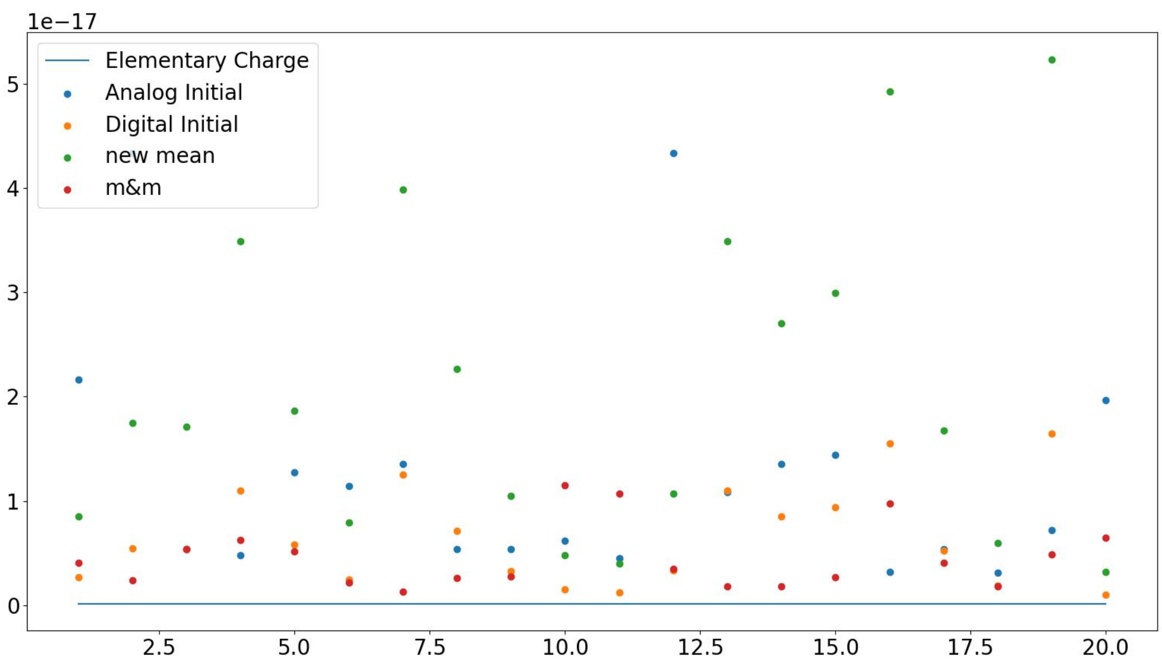

Figure 6: Digital Initial Trial Data

5 Discussion and Conclusions

In conclusion, based on data gathered throughout this project, the Millikan Experiment as conducted in a academic laboratory setting can be increased in its accuracy and its effectiveness. The calculated data became much closer to theoretical value with a smaller distribution. Although the values of computed charge didn't get as small as hoped, it's clear why that wasn't possible. This can be explained with the help of the error analysis shown in section 2, showing that selecting a drop with a high fall rate and high stopping voltage would yield a smaller computed charge. This makes sense because the droplets wouldn't be heavy enough to experience severe effects by gravity, but also not heavy enough to have been clumped together. The high stopping voltage would ensure that it has gained a sufficient amount of electrons. All of this means that the drop would have to be small and carrying a lot of charge. With smaller drops, they are less likely to gather as much charge through the atomizer than that of a bigger drop. I attempted to gather data with a small drop with a high stopping voltage, but with the equipment setup that we currently have, I was not able to do so. The inability to replicate this in the lab shows why the experimental data can become more accurate, but not more precise to the value of the elementary charge of an electron.

Future experiments with this apparatus should focus on applying the "Measure Measure" approach. This would involve the data collector holding a stopwatch in one hand, and controlling the voltage output with the other. The selected drop should first be measured for its fall rate moving from one of the scope's reticle lines to the other—which are $0.5mm$ apart. After getting the time, and not taking eyes away from the particle being measured, the voltage needs to be turned on and adjusted until the particle is suspended in the same spot for roughly 30 seconds. The voltage displayed on the voltmeter and the time displayed on the stopwatch should both be written down before moving on to measure the next droplet.

The increased accuracy of the experiment that this project created makes it possible to reintroduce it into the laboratory setting. Before, the original experiment called for 50 particles to be measured. As the collected data is now more accurate, less data trials could be taken. The experiment would now be less painful for students to perform, and provide better data for conceptual analysis.

6 Bibliography

Yingzi Li et al 2015 Eur. J. Phys. 36 055022