The Drag Coefficient and Air-Resistance:

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Introduction:

The aim of this project is to find the best model of air resistance on a projectile, in the shape of a short cylinder traveling along its axis. Three commonly used models in undergraduate level physics classes will be examined: a vacuum model that ignores drag, a linear model that assumes the drag force is proportional to the velocity, and a quadratic model that assumes the drag force is proportional to the square of the velocity.

To select the best model, a potato cannon was be used to fire cylindrical potatoes and their range was compared to predictions of the models. Statistical analysis was performed on the difference between the predictions and the actual ranges.

Theory:

Drag is a force that results from an object with some cross-section moving through a viscous fluid. Drag generated by air is commonly referred to as air resistance. Drag can be difficult to incorporate into projectile problems. For this reason, most introductory level physics classes choose to ignore it as if the projectile was in a vacuum. If this model is used, the range of the projectile is determined by the following formula:

$$x(t) = x_0 + \frac{v_{x0}}{g} \left(v_{y0} + \sqrt{v_{y0}^2 + 2gy_0} \right)$$

Once drag is considered, the next simplest model to use is a linear relationship between the force and the velocity. The drag force then will be as follows:

$$\vec{F}_{drag} = -\mu_l \vec{v}$$

The constant, μ_l , in this equation is determined by properties of the fluid and projectile. In this experiment the value will be estimated to best fit the actual ranges. Using this model the range is then:

$$x(t) = \frac{mv_{x0}}{\mu_l} \left[1 - e^{-\frac{v_{y0}}{\frac{mg}{\mu_l} + v_{y0}} \pm \sqrt{\frac{v_{y0}^2 + 2gy_0 + \frac{2\mu_l v_{y0} y_0}{m}}{\left(\frac{mg}{\mu_l} + v_{y0}\right)^2}}} \right]$$

Another commonly used model is one that uses a quadratic relationship between the velocity and the force. The drag force will be as follows:

$$\vec{F}_{drag} = -\mu_q v^2 \hat{v}$$

The coefficient, μ_q , for this model also is determined by properties of the fluid and the projectile; however, there is a known formula for this constant:

$$\mu_q = \frac{C_D \rho A}{2}$$

Where ρ is the density of the fluid, A is the cross-sectional area, and C is the drag coefficient which must be determined experimentally. The constant, μ_q , will be estimated in this project to best fit the experimental data. Unfortunately, this model leads to a pair of linked differential equations that cannot be solved analytically. Instead numerical strategies must be used to project ranges. One of the easiest methods to use is Euler's Method, and it can be executed using Microsoft Excel or other mathematical software. Euler's Method was used to generate the quadratic projections in this experiment.

Once these predictions and experimental data have been collected, they can be compared using the F-test. The F-statistic compares the standard deviations of two samples, and when compared to a critical value determined by the F-distribution, will determine whether or not the differences in the standard deviations are statistically significant.

Derivations of the range projections for each model as well as an explanation of the use and meaning of the F-test in this kind of application are contained in the appendix.

Methods:

This experiment will be conducted by shooting a cylindrical potato two inches in diameter and three inches long from a potato-cannon at a known initial velocity and angle of trajectory. The potato will have known cross-sectional area, length, and mass. Additionally the wind velocity and direction will be known, as well as the relative elevation of the potato at launch and landing. The distance from launch where the potato lands (the range) will be measured and recorded.

The cannon that was used has a two inch diameter, rifled barrel to ensure the projectile will maintain flight along its axis without wobbling, which would change the cross-sectional area during flight. It will be stable to maintain orientation and angle of trajectory. A photo-gate was attached to the end of the barrel which was used to calculate the initial velocity of the projectile. The cannon is pneumatic, and utilized a sprinkler valve to release the pressure quickly and evenly. Photographs of the cannon can be found in the appendix.

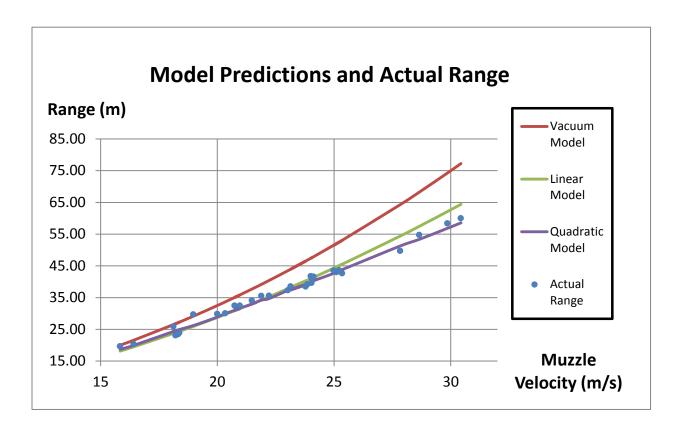
The potatoes were then cut by pushing them through two inch PVC pipe to ensure proper fit into the barrel. The ends were then cut so they are each three inches long. They were then weighed to find the mass.

The field was measured using a hose-end level to determine the elevation at the landing areas. An anemometer will be used to gauge the wind speed and direction during the test. Thirty shots were fired and the time registered by the photo-gate, the distance from the launch point at which it lands, and all the previously mentioned data were recorded.

After the experiment is conducted the data for each shot will be used to calculate projections in all three models. Then the actual data will be subtracted from each corresponding projection to give a set of differences between projection and data point for each model. Then the mean and standard deviation of each set of differences will be calculated, which will allow for the F-test to be used to compare the accuracies of each model against each other.

Results:

Below is a graph of the collected data points and the projections of each model. For these predictions $\mu_l=0.011$, and $\mu_q=0.0014$. The quadratic predictions were created using a time-step of five hundredths of a second.



Qualitatively, it is clear the vacuum predictions are well above the actual ranges. Both the linear and quadratic models are fairly close to the actual data points, but the quadratic model looks more accurate overall. However, it appears the linear model appears to diverge from the actual ranges as the muzzle velocity increases, while the quadratic model seems to remain close to the actual data. At lower muzzle velocities the linear and quadratic models look equally close to the actual data, but the quadratic model appears to be much closer at high muzzle velocities.

For a quantitative inspection, the average and standard of the differences between the models and actual ranges must be calculated. Those values can be found in the table below.

	Vacuum - Actual	Linear - Actual	Quadratic - Actual
Mean	6.29	0.48	-0.47
Standard Deviation	4.46	1.91	1.11

The F-test can be performed on each combination of the three models. The names have been edited for space, but they are the relationships between the differences of predictions to actual data points, not the predicted values themselves.

	Vacuum vs Linear	Vacuum vs Quadratic	Linear vs Quadratic
F	5.432	16.245	2.990
F_crit (90% confidence)	1.620	1.620	1.620

The proper way to interpret results of an F-test is explained in the appendix. For this test, the first model in the column name is the first sample used and the second name corresponds to the second sample. The critical value represents a ninety percent confidence level.

There is another set of F-test results that are revealing. For this set only the linear and quadratic differences are being compared, but the thirty shots have been split into three groups: The first contains the ten lowest muzzle velocities, the second contains the ten middle muzzle velocities, and the third contains the ten highest muzzle velocities. The same F-test from the third column above is then performed for each of the three groups individually.

	Group 1 (L vs Q)	Group 2 (L vs Q)	Group 3 (L vs Q)
F	0.932	2.398	2.728
F_crit (85% confidence)	0.629	2.050	2.050
F_crit (90% confidence)	0.410	2.440	2.440

For this test, the first model in the column name is the first sample used and the second name corresponds to the second sample. The critical value represents a ninety percent confidence level.

Discussion of Results:

Some qualitative results have already been discussed above; however their quantitative evidence has only been presented in a raw form. The first observation was that the vacuum model was very inaccurate; far worse than either the quadratic or linear models. As seen in the above table, the F-test with ninety percent confidence agrees that it is less accurate. Upon further investigation the F-test actually agrees with that assertion with up to 99.99% confidence (F_crit (99.99%) = 4.261).

The second observation was that while both the linear and quadratic models were fairly accurate overall, the quadratic model was more accurate. Again the F-test with ninety percent confidence agrees with this observation. Upon further investigation the F-test actually agrees with that assertion with up to ninety-nine percent confidence (F_crit (99%) = 2.423).

The third observation was that the linear model was just as accurate as the quadratic model at low muzzle velocities, but at the high muzzle velocities the quadratic model was more accurate. The third table agrees with this statement with ninety percent confidence at the low and high groups (groups 1 and 2); that is the two models are just as accurate for the low muzzle velocity group, but the quadratic model is more accurate for the high velocity group. The middle group (group 2), would agree with the statement the linear model is less precise than the quadratic with eighty-five percent confidence, but would not agree at the ninety percent confidence level. The calculated F values would indicate that the linear model becomes less accurate than the quadratic model as muzzle velocity increases. Changing the confidence level of the F-test only changes the maximum muzzle velocity the linear model will give an acceptable projection of the range.

Conclusions:

These results would indicate that of the three models examined, the quadratic model is the best estimate of the drag force on an object in a fluid. However, for certain applications, it may be appropriate to model the drag force using a linear model because it can be solved for analytically. This should only be done at low initial velocities.

There are lots of situations that need further examination. Different shapes of the object in question, more extreme velocities, and different fluids all impact the drag coefficients and therefore may change the accuracy of the models. Additionally, since the quadratic model is generated using Euler's Method, the predictions can be impacted by using a different time-step. Any of these changes could be tested using a similar experiment.

Appendix

Contents:

- 1. Derivation of the vacuum range
- 2. Derivation of the linear range
- 3. Estimation of the quadratic range
- 4. Explanation of the F-test
- 5. Photographs and details of the potato cannon

This is a derivation of the range formula of a projectile travelling through a vacuum. We start with Newton's Second Law in the horizontal direction. There is no force acting in the horizontal direction on the projectile, so the acceleration in the horizontal direction is zero.

$$F_x = ma_x = 0$$

Knowing the velocity is a constant, we can manipulate the definition of velocity to find a function of horizontal position.

$$v_{xo} = \frac{x(t) - x_0}{t}$$

$$x(t) - x_0 = v_{xo}t$$

$$x(t) = x_0 + v_{x0}t$$

Now we turn our attention to the vertical direction. Again we start with Newton's Second Law.

$$F_{y} = ma_{y} = -F_{g} = -mg$$
$$a_{y} = -g$$

We use this value in one of the basic kinematic relations.

$$x_f - x_i = \frac{1}{2} a_x t^2 + v_x t$$

$$y(t) = -\frac{g}{2}t^2 + v_{y0}t + y_0$$

We now set the equation equal to zero and solve for time using the quadratic formula.

$$y(t) = 0 = -\frac{1}{2}gt^{2} + v_{y0}t + y_{0}$$

$$t = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$t = \frac{-(v_{y0}) \pm \sqrt{(v_{y0})^{2} - 4(-\frac{g}{2})(y_{0})}}{2(-\frac{g}{2})}$$

$$t = \frac{v_{y0} \pm \sqrt{v_{y0}^{2} + 2gy_{0}}}{a}$$

Now we insert these values for t into the horizontal equation to find our range equation.

$$x(t) = x_0 + v_{x0} \left(\frac{v_{y0} \pm \sqrt{v_{y0}^2 + 2gy_0}}{g} \right)$$

$$x(t) = x_0 + \frac{v_{x0}}{g} \left(v_{y0} \pm \sqrt{v_{y0}^2 + 2gy_0} \right)$$

This is a derivation of the range formula of a projectile considering linear air resistance. First the definition of the linear frictional force:

$$F_{linear} = C_l \eta l v = \mu_l v$$

We begin by writing Newton's Second Law in the horizontal direction.

$$F_x = ma_x = -F_{Dx} = -\mu_l v_x$$

We now divide by the mass and rewrite the acceleration as the derivative of the velocity.

$$a_x = \frac{dv_x}{dt} = -\frac{\mu_l}{m}v_x$$

Now we separate variables and integrate.

$$\int_0^t dt = -\frac{m}{\mu_l} \int_{v_{x0}}^{v_x} dv_x / v_x$$

$$t - 0 = -\frac{m}{\mu_l} [\ln v_x - \ln v_{x0}] = -\frac{m}{\mu_l} \ln \frac{v_x}{v_{x0}}$$

We now manipulate the formula to isolate the horizontal velocity.

$$t = -\frac{m}{\mu_l} \ln \frac{v_x}{v_{x0}}$$
$$-\frac{\mu_l}{m} t = \ln \frac{v_x}{v_{x0}}$$
$$\frac{v_x}{v_{x0}} = e^{-\frac{\mu_l}{m}t}$$
$$v_x = v_{x0} e^{-\frac{\mu_l}{m}t}$$

We integrate once more to find the horizontal position of the projectile.

$$\int_{0}^{t} v_{x} dt = v_{x0} \int_{0}^{t} e^{-\frac{\mu_{l}}{m}t} dt$$

$$x(t) - x(0) = -\frac{mv_{x0}}{\mu_{l}} \left[e^{-\frac{\mu_{l}}{m}t} - 1 \right]$$

$$x(t) = \frac{mv_{x0}}{\mu_{l}} \left[1 - e^{-\frac{\mu_{l}}{m}t} \right]$$

We now turn our attention to the vertical motion. We begin with Newton's Second Law again.

$$F_{\nu} = ma_{\nu} = -F_{a} - F_{D\nu} = -mg - \mu_{l}v_{\nu}$$

We divide the mass over and rewrite the acceleration as the derivative of the velocity.

$$\frac{dv_y}{dt} = -g - \frac{\mu_l}{m}v_y$$

Next we separate variables and integrate.

$$\int_0^t dt = -\int_{v_{y_0}}^{v_y} dv_y / g + \frac{\mu_l}{m} v_y$$

$$t - 0 = -\frac{m}{\mu_l} \left[\ln \left(g + \frac{\mu_l}{m} v_y \right) - \ln \left(g + \frac{\mu_l}{m} v_{y_0} \right) \right]$$

$$t = \frac{m}{\mu_l} \ln \left(\frac{g + \frac{\mu_l}{m} v_{y_0}}{g + \frac{\mu_l}{m} v_y} \right)$$

Now we manipulate the equation to isolate the velocity.

$$t = \frac{m}{\mu_l} \ln \left(\frac{g + \frac{\mu_l}{m} v_{y0}}{g + \frac{\mu_l}{m} v_y} \right)$$

$$\frac{\mu_l}{m} t = \ln \left(\frac{g + \frac{\mu_l}{m} v_{y0}}{g + \frac{\mu_l}{m} v_y} \right)$$

$$\frac{g + \frac{\mu_l}{m} v_{y0}}{g + \frac{\mu_l}{m} v_y} = e^{\frac{\mu_l}{m} t}$$

$$g + \frac{\mu_l}{m} v_y = \left(g + \frac{\mu_l}{m} v_{y0} \right) e^{-\frac{\mu_l}{m} t}$$

$$\frac{\mu_l}{m} v_y = \left(g + \frac{\mu_l}{m} v_{y0} \right) e^{-\frac{\mu_l}{m} t} - g$$

$$v_y = \left(\frac{mg}{\mu_l} + v_{y0} \right) e^{-\frac{\mu_l}{m} t} - \frac{mg}{\mu_l}$$

Now we integrate to find the vertical position.

$$\int_{0}^{t} v_{y} dt = \frac{mg}{\mu_{l}} \int_{0}^{t} e^{-\frac{\mu_{l}}{m}t} dt + v_{y0} \int_{0}^{t} e^{-\frac{\mu_{l}}{m}t} dt - \frac{mg}{\mu_{l}} \int_{0}^{t} dt$$
$$y(t) - y(0) = -\frac{m^{2}g}{\mu_{l}^{2}} \left(e^{-\frac{\mu_{l}}{m}t} - 1 \right) - \frac{mv_{y0}}{\mu_{l}} \left(e^{-\frac{\mu_{l}}{m}t} - 1 \right) - \frac{mg}{\mu_{l}} t$$

$$y(t) = \frac{m^2 g + m\mu_l v_{y0}}{\mu_l^2} \left(1 - e^{-\frac{\mu_l}{m}t} \right) - \frac{mg}{\mu_l} t + y_0$$

Now we will find the times at which the height is zero. One of these times will correspond to the point where the projectile lands. First we set the equation equal to zero.

$$y(t) = \frac{m^2 g + m\mu_l v_{y0}}{\mu_l^2} - \frac{m^2 g + m\mu_l v_{y0}}{\mu_l^2} e^{-\frac{\mu_l}{m}t} - \frac{mg}{\mu_l}t + y_0 = 0$$

Now we use a Taylor Expansion to estimate the exponential term.

$$f(t) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (t-a)^n$$

$$e^{-\frac{\mu_l}{m}t} = 1 - \frac{\mu_l}{m}t + \left(\frac{\mu_l}{m}\right)^2 \frac{t^2}{2} - \left(\frac{\mu_l}{m}\right)^3 \frac{t^3}{6} + \left(\frac{\mu_l}{m}\right)^4 \frac{t^4}{24} - \left(\frac{\mu_l}{m}\right)^5 \frac{t^5}{120} \dots$$

We will only use the first three terms here. We replace the estimation for the exponential term and reduce the equation.

$$0 = \frac{m^2 g + m\mu_l v_{y0}}{\mu_l^2} - \frac{m^2 g + m\mu_l v_{y0}}{\mu_l^2} \left[1 - \frac{\mu_l}{m} t + \left(\frac{\mu_l}{m}\right)^2 \frac{t^2}{2} \right] - \frac{mg}{\mu_l} t + y_0$$

$$0 = \frac{mg + \mu_l v_{y0}}{\mu_l} t - \frac{mg + \mu_l v_{y0}}{2m} t^2 - \frac{mg}{\mu_l} t + y_0$$

$$0 = -\frac{mg + \mu_l v_{y0}}{2m} t^2 + v_{y0} t + y_0$$

Now we use the Quadratic Equation to solve for the zeroes.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-v_{y0} \pm \sqrt{v_{y0}^2 - 4\left(-\frac{mg + \mu_l v_{y0}}{2m}\right)(y_0)}}{2\left(-\frac{mg + \mu_l v_{y0}}{2m}\right)}$$

$$t = \frac{mv_{y0}}{mg + \mu_l v_{y0}} \pm \sqrt{\frac{m^2 v_{y0}^2}{(mg + \mu_l v_{y0})^2} + \frac{m^2}{(mg + \mu_l v_{y0})^2}} \frac{2(mg + \mu_l v_{y0})y_0}{m}$$

$$t = \frac{mv_{y0}}{mg + \mu_l v_{y0}} \pm \sqrt{\frac{m^2 v_{y0}^2}{(mg + \mu_l v_{y0})^2} + \frac{2my_0}{mg + \mu_l v_{y0}}}$$

$$t = \frac{mv_{y0}}{mg + \mu_l v_{y0}} \pm \sqrt{\frac{m^2 v_{y0}^2 + 2my_0 (mg + \mu_l v_{y0})}{(mg + \mu_l v_{y0})^2}}$$

Finally we insert these values of t into the horizontal equation to find the range. First we will only examine the exponent.

$$-\frac{\mu_{l}v_{y0}}{mg + \mu_{l}v_{y0}} \pm \sqrt{\frac{\mu_{l}^{2}}{m^{2}} \left(\frac{m^{2}v_{y0}^{2} + 2my_{0}(mg + \mu_{l}v_{y0})}{(mg + \mu_{l}v_{y0})^{2}}\right)}$$

$$-\frac{v_{y0}}{mg} \pm \sqrt{\frac{v_{y0}^{2} + 2gy_{0} + 2\frac{\mu_{l}v_{y0}}{m}y_{0}}{\left(\frac{mg}{\mu_{l}} + v_{y0}\right)^{2}}}$$

$$x(t) = \frac{mv_{x0}}{\mu_{l}} \left(1 - e^{-\frac{v_{y0}}{mg} + v_{y0}}\right)^{\frac{1}{2}} \sqrt{\frac{v_{y0}^{2} + 2gy_{0} + \frac{2\mu_{l}v_{y0}y_{0}}{m}}{\left(\frac{mg}{\mu_{l}} + v_{y0}\right)^{2}}}$$

This is a method for estimating the range of a projectile considering quadratic air resistance. First, the definition of the quadratic frictional force:

$$\vec{F}_{Quadratic} = -\frac{C_D \rho A}{2} v^2 \hat{v} = -\mu_q v^2 \hat{v}$$

We begin by writing Newton's Second Law as a vector equation.

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2} = \vec{F}_g - \vec{F}_{drag} = m\vec{g} - \mu_q v^2 \hat{v} = m\vec{g} - \mu_q v \vec{v}$$

Now we insert the appropriate vectors and divide by the mass.

$$\frac{d^2}{dt^2} \binom{x(t)}{y(t)} = \binom{0}{-g} - \frac{\mu_q \sqrt{v_x^2 + v_y^2}}{m} \binom{v_x}{v_y}$$

Next we can separate the two equations.

$$\frac{dv_x}{dt} = a_x = -\frac{\mu_q \sqrt{v_x^2 + v_y^2}}{m} v_x$$

$$\frac{dv_{y}}{dt} = a_{y} = -g - \frac{\mu_{q} \sqrt{v_{x}^{2} + v_{y}^{2}}}{m} v_{y}$$

As seen above the two equations are linked, and cannot be solved analytically. Using a numerical method we can estimate the range. We will use Euler's Method here. The calculation will be performed using Microsoft Excel, but the strategy will be presented below.

Conceptually, if we break the motion of the projectile into time steps we can estimate the acceleration, velocity, and position for each direction over each time step using the definitions of velocity and acceleration as well as initial conditions.

We need six formulas for our estimation, the two acceleration equations we found earlier. We will rewrite them for our application.

$$a_x^n = -\frac{\mu_q \sqrt{(v_x^n)^2 + (v_y^n)^2}}{m} v_x^n$$

$$a_{y}^{n} = -g - \frac{\mu_{q}\sqrt{(v_{x}^{n})^{2} + (v_{y}^{n})^{2}}}{m}v_{y}^{n}$$

We will now find the equations for the velocities.

$$a = \frac{v_f - v_i}{t_f - t_i}$$

$$v_f - v_i = a(t_f - t_i)$$

$$v_f = v_i + a(t_f - t_i)$$

We rename the variables for our application.

$$v_x^n = v_x^{n-1} + a_x^{n-1}(t^n - t^{n-1})$$
$$v_y^{n+1} = v_y^{n-1} + a_y^{n-1}(t^n - t^{n-1})$$

Finally, we find the positions.

$$v = \frac{x_f - x_i}{t_f - t_i}$$
$$x_f - x_i = v(t_f - t_i)$$
$$x_f = x_i + v(t_f - t_i)$$

We rename the variables for our application.

$$x^{n} = x^{n-1} + v_{x}^{n-1}(t^{n} - t^{n-1})$$
$$y^{n} = y^{n-1} + v_{y}^{n-1}(t^{n} - t^{n-1})$$

Now that we have our formulas we insert them into an Excel table like the one below, appropriately referencing cells. In the time column we use a time step of five hundredths of a second, but that can be freely manipulated. The smaller the step, the more accurate the result will be.

time	V_x	V_y	х	у	A_x	A_y
$t^0 = 0$	initial	initial	0	initial	$= -\frac{\mu_q \sqrt{(v_x^n)^2 + (v_y^n)^2}}{m} v_x^n$	$= -g$ $-\frac{\mu_q \sqrt{(v_x^n)^2 + (v_y^n)^2}}{m} v_y^n$
$= t^{n-1} $ $+ .05$	$ \begin{vmatrix} = v_x^{n-1} \\ + a_x^{n-1} (t^n \\ - t^{n-1}) \end{vmatrix} $	$= v_y^{n-1} + a_y^{n-1}(t^n - t^{n-1})$	$ \begin{vmatrix} = x^{n-1} \\ + v_x^{n-1} (t^n \\ - t^{n-1}) \end{vmatrix} $	$ \begin{vmatrix} = y^{n-1} \\ + v_y^{n-1} (t^n \\ - t^{n-1}) \end{vmatrix} $		
	$-t^{n-1}$	$-t^{n-1}$)	$-t^{n-1}$	$-t^{n-1}$)		

Once the formulas are inserted and appropriately referenced, the formulas can be dragged down simultaneously. We do this until the first time the vertical position (y column) is negative. We then average the last two horizontal positions (x column) and that result is our estimation of the range.

This is an explanation of the F-test, proper interpretation, and its use in model comparison.

The F-test is a statistical test that compares the standard deviations of two data sets and determines whether or not they are statistically different. The definition of the F-statistic is as follows:

$$F = \frac{s_1^2}{s_2^2}$$

The s's are the standard deviations of each sample. If the standard deviations are the same, we would expect an F-value of one. Depending on which standard deviation is larger, F-values can be any real, positive number.

The F-test is the comparison of this calculated value to a value determined by the F-distribution, usually referred to as the critical value. This value requires knowing the degrees of freedom of each data set, one less than the number of data points in a set, and the confidence level to which the test will be performed. Critical values can be found in tables, or by using the F.INV or F.INV.RT commands in Microsoft Excel. If the F-value being considered is less than one, the F critical value should be less than one. Likewise, if the F-value being considered is greater than one, the F critical value should also be greater than one.

The interpretation of this test is simple. If the F-value is less than one and the smaller F critical value, the two standard deviations are not equal. If the F-value is greater than the F critical value the two standard deviations are statistically the same. If the F-value is greater than one and greater than the F critical value, the two standard deviations are not equal. If the F-value is less than the F critical values the two standard deviations are statistically the same.

Using the F-test can determine the relative accuracy of models by performing it on the difference between the projections and actual data. If the standard deviation of one projection is determined to be larger than another, the former is less accurate than the latter.

This is the potato cannon used in the experiment. It has a rifled barrel three feet long and two inches in diameter. The end of the barrel is two feet above the ground. There is a photo gate attached to the end of the barrel to measure the velocity of the shot. The cannon is pneumatic, and is pressurized using the Schrader valve at the end of the pressure reservoir. Then there is a button used to open the electric sprinkler valve, which releases the pressure quickly and evenly into the barrel, ejecting the potato.

