Alpha Particle Scattering Rate Dependence on Material Thickness Using the Rutherford Scattering Model

Joshua Striano
Faculty Advisor: Dr. David Gore

April 2024

1 Abstract

This capstone was to verify that the rate at which α-particles scatter off of gold foil is directly proportional to the thickness of the gold (Eq. 8). A 0.1 μCi polonium-210 source produces α-particles which are directed at sheets of gold foil layered on a plastic base. A photo-multiplier tube (PMT) with a zinc sulfide sheet attached to the front is then used to detect α-particles that scatter off the gold at angles between 85° and 130°. The signals from the PMT are then processed before a counting module and stopwatch are used to determine the rate at which α-particles scatter. Data was taken for up to 12 sheets and the rates plotted. Collision theory predicts that this plot will be linear.

2 Introduction

In the early 1900s, Geiger and Marsden, under the supervision of Ernest Rutherford, began studying large angle scattering of α-particles. One of the principles they investigated was how variations in the thickness of the target affected scattering rate. They found that the rate of scattering increased linearly with the amount of foil. This suggests that scattering happens within the volume of the material instead of solely on the surface.

The purpose of this capstone is to replicate the results of Geiger and Marsden and compare them to the theory for Rutherford scattering (Eq. 8). To achieve this, I created an apparatus for detecting α-particles and measured the rate of scattering for varying thickness of foil. I then fitted my data and computed a χ^2 per degree of freedom as a goodness of fit. Finally, I compared the fit to what is expected by the Rutherford scattering model.

3 Theory

3.1 Rutherford Scattering

The phenomenon this capstone is studying is Rutherford Scattering. Which is non-relativistic, elastic scattering where the only interaction between the particle and target is the coulomb force. The particles will only be scattered if they come close enough to the gold nuclei for the coulomb force to have a significant effect. This can be thought of as the effective size of the target nuclei. This size is the effective cross sectional area and given the letter σ. Given the effective "size" and the number of target nuclei per unit area, n_{tar}, the probability of a single α-particle being scattered is

$$\Pi(Hit) = \frac{n_{tar} a \sigma}{a} = n_{tar}\sigma \tag{1}$$

The target density per unit area can be replaced with a volumetric target density n and the thickness of the material L using

$$n_{tar} = nL \tag{2}$$

For a beam containing N_i α-particles, the number of scattered particles, N, will be

$$N = N_i n L \sigma \tag{3}$$

Eq. 3 describes the total number of particles scattered. These are scattered over the entire solid angle. To figure out how many reach the detector, it is needed to figure out how many scatter into that portion of the solid angle, $d\Omega$. The portion of the cross-section that scatters into some part of the solid angle is

$$d\sigma = \frac{d\sigma(\theta, \phi)}{d\Omega} d\Omega \tag{4}$$

where $\frac{d\sigma(\theta,\phi)}{d\Omega}$ is called the differential cross-section. Thus, the number of detected particles is given by

$$N = N_i n L \frac{d\sigma(\theta, \phi)}{d\Omega} d\Omega \tag{5}$$

For Rutherford scattering, the differential cross-section is given by the formula

$$\frac{d\sigma(\theta, \phi)}{d\Omega} = \left(\frac{Z Z' e^2}{4\pi\epsilon_0} \frac{1}{4T} \right)^2 \frac{1}{\sin^4(\theta/2)} \tag{6}$$

where Z is the target's atomic number, Z' is the projectile's atomic number, e is the charge of a proton, ϵ_0 is the permittivity of free space, T is the projectile's kinetic energy, and θ is the angle between incident beam direction and detector. Then the solid angle subtended by a detector of area A and a distance r from the target is

$$d\Omega = \frac{A}{r^2} \tag{7}$$

Thus, for Rutherford scattering the number of α-particles detected is

$$N(L) = N_i n L \frac{A}{r^2} \frac{Z^2 e^4}{16\pi^2 \epsilon_0^2} \frac{1}{4T^2} \frac{1}{sin^4(\theta/2)} \tag{8}$$

Thus, the number of detected particles should increase linearly with the thickness of the material.

3.2 Thickness Analysis

The gold foil is too thin to measure thickness directly, therefore it needs to be calculated from the mass, m, of each sheet. Using the definition of density, ρ

$$\rho = \frac{m}{V} \tag{9}$$

The volume can be rewritten in terms of the area, A, and the thickness, L, which then gives density as

$$\rho = \frac{m}{AL} \tag{10}$$

Solving for the thickness

$$L = \frac{m}{\rho A} \tag{11}$$

Each sheet of gold has to be attached to a holder for it to stand upright. Thus, the mass of the gold can be calculated by measuring the change in mass of the apparatus after applying the gold.

$$m_{gold} = m_2 - m_1 \tag{12}$$

where m_1 is the mass before the gold was applied and m_2 was the mass after. Combining eq. 11 and eq. 12 gives the thickness as

$$L = \frac{m_2 - m_1}{\rho A} \tag{13}$$

4 Methods

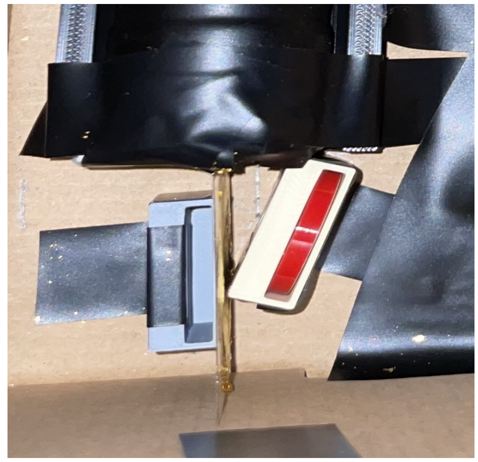
4.1 Construction

The construction started with modeling and 3D printing a small box to hold the polonium source. The box has dimensions of 2.8 cm wide by 2.4 cm tall by 10 cm long. The front 0.5 cm is solid plastic with a hole in the center to collimate the beam. The box originally had a hole with a diameter of 1 mm but was changed to 6 mm to allow more α-particles through.

The use of a PMT required the set up to be light-tight. To achieve this, a shoe box that was wrapped in black electrical tape was used. The shoe box had to have holes cut in it for the power and signal wire of the PMT. The PMT was then placed into the shoe box. The holes for the wires were then taped over to ensure no light could enter. Next, a support was modeled and 3D printed to hold the PMT. This support was 4.8 cm wide, 2 cm tall, and 10 cm long with a 3.8 cm diameter hole cut through it. A zinc-sulfide screen was taped to the front of the PMT and half of it was covered to ensure no α-particle could strike the screen without scattering first.

Each sheet of gold foil used had an area of 4 cm by 4 cm so a piece of hard plastic that was 4.5 cm in length by 6 cm in height was cut out to hold them. This was then taped to the original 3D printed box with the 1 mm hole. To adhere gold to this, each 8 cm by 8 cm sheet of gold foil was cut into 4 equal sections. Then, one quarter of the sheet would be laid out and a pair of tweezers was used to hold down one side of the foil. The base of the plastic would be lined up with the foil before being pressed on to the gold and having it is edges smoothed.

(a) Construction (Overview)



(b) Construction (Focused)

Figure 1: Construction

4.2 Execution

To characterize the PMT, data was taken of the background rate measured at different applied voltages. Additionally, data was taken on the background rate throughout the day. For this, the PMT was set at one voltage and the count was recorded every hour.

For the run data, the mass of each sheet was measured such that the thickness could be calculated from the density and mass of the gold. The rate of α-particles scattering was then measured using a counting module and a stopwatch.

5 Data

5.1 PMT Voltages vs Background Count Rate

Voltage (V)	Counts (over 5 minutes)
1700	14
1730	14
1760	11
1790	21
1820	53
1850	111

Table 1: Counts at Different Voltages

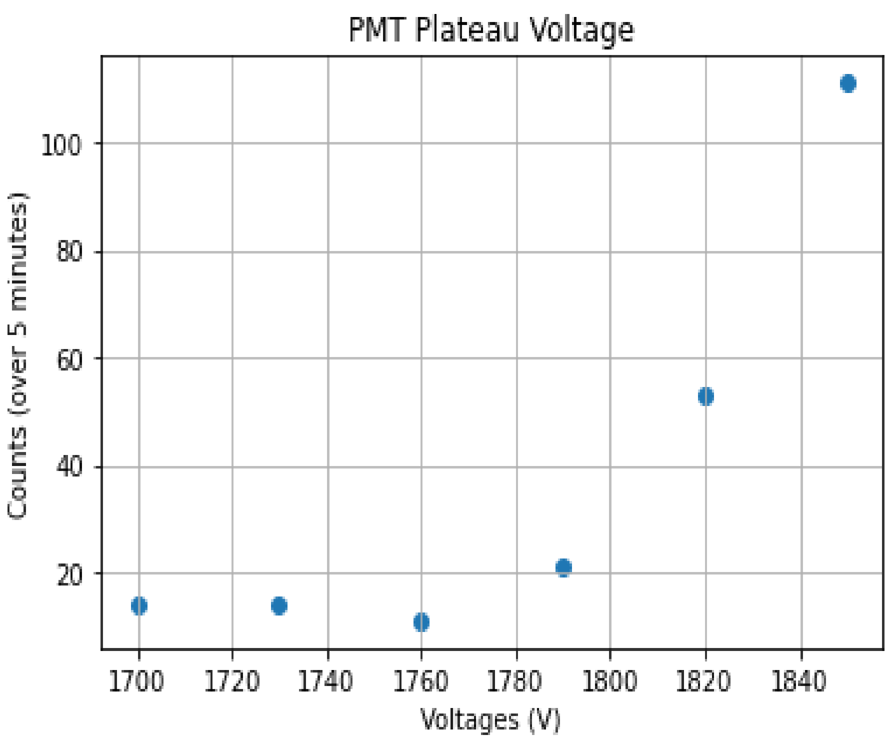
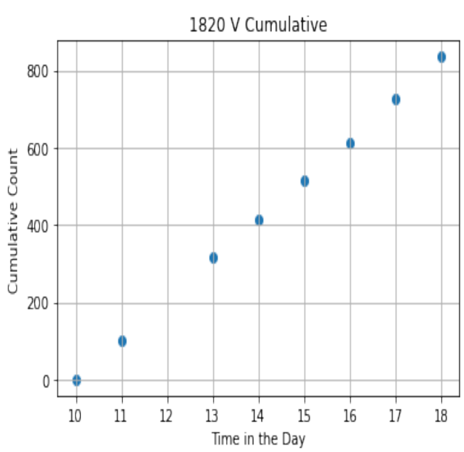


Figure 2: Counts at Different Voltages

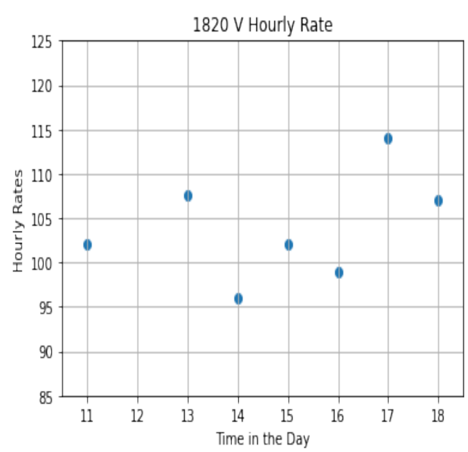
This data was collected with the source present, however, due to the configuration of my setup, most of the counts are from background. From these results, I decided to use a 1750 V supply to limit the number of background signals detected since at higher voltages, the detector will measure lower energy background signals.

Time	Cumulative Counts
10 am	0
11 am	102
1 pm	317
2 pm	413
3 pm	515
4 pm	614
5 pm	728
6 pm	835

Table 2: Counts Throughout the Day



(a) Background Counts (Cumulative)



(b) Background Counts (Hourly)

Figure 3: Background Throughout the Day

5.2 Run Data

Sheet	Mass (g)	Time (min)	Counts	Rate (min^{-1})
Background	N/A	299	93	0.311
0	4.75	195	70	0.359
1	4.76	200	84	0.42
1	4.76	180	77	0.428
2*	4.76	180	184	1.022
2	4.76	196	74	0.378
3*	4.77	180	304	1.689
3	4.77	180	75	0.417
4*	4.77	180	371	2.061
4	4.77	180	87	0.483
5	4.77	180	112	0.622
5*	4.77	180	1701	9.45
5	4.77	180	106	0.589
6	4.78	780	405	0.519
7	4.78	180	121	0.672
8	4.78	180	90	0.5
9*	4.79	180	1076	5.978
9	4.79	180	95	0.5278
10	4.79	180	102	0.567
11	4.79	181	103	0.5691
12	4.79	180	118	0.656

Table 3: Mass and Count Rate Per Sheet

Asterisk indicate runs that were not include in the final data as they had an unusually high number of erroneous signals. These signals were caused by vibrations in the building due to nearby construction.

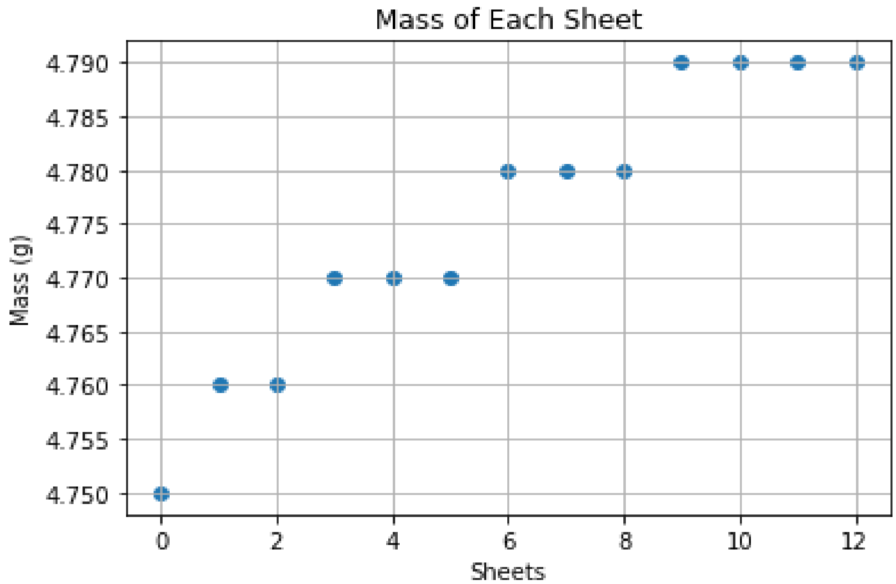


Figure 4: Mass Per Sheet

Fig. 4 shows how the mass of the holder changes with each sheet applied. The thickness of each sheet could be calculated using eq. 13. However, this was not done as the scale did not give good thickness resolution.

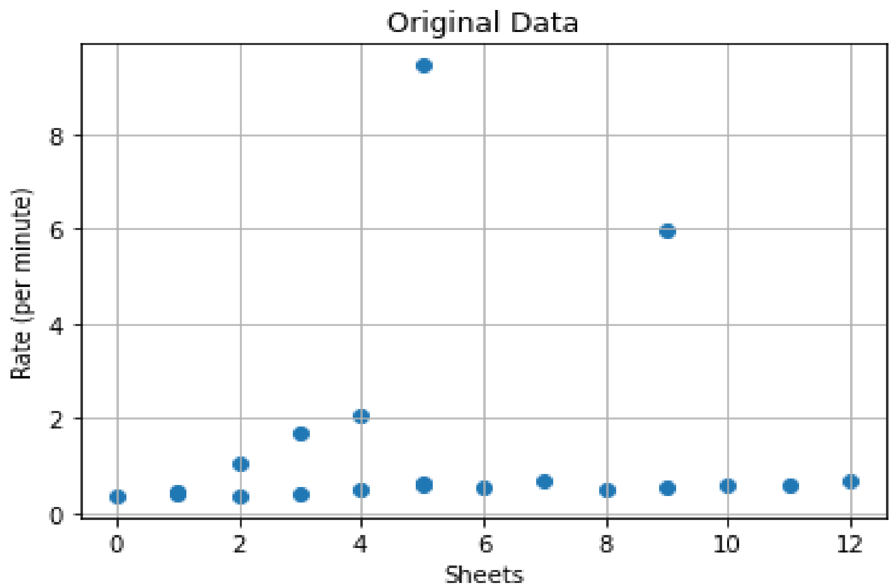


Figure 5: Original Data

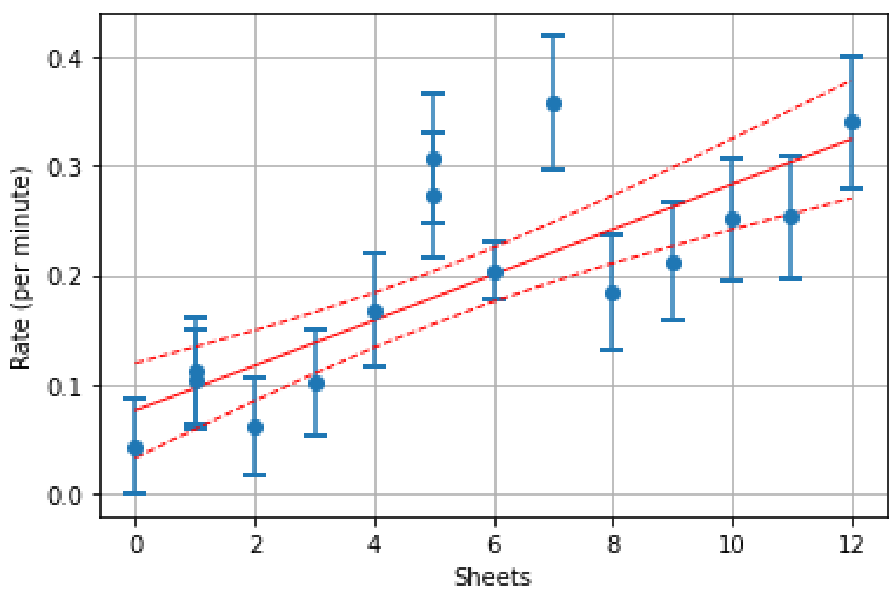


Figure 6: Fitted Data with Error

The above graph is without the rates that had an unusually high number of erroneous signals. The fit has a slope of $0.020 \pm 0.003\ \frac{1}{\text{min sheet}}$ and a χ^2 per degree of freedom of 2.043.

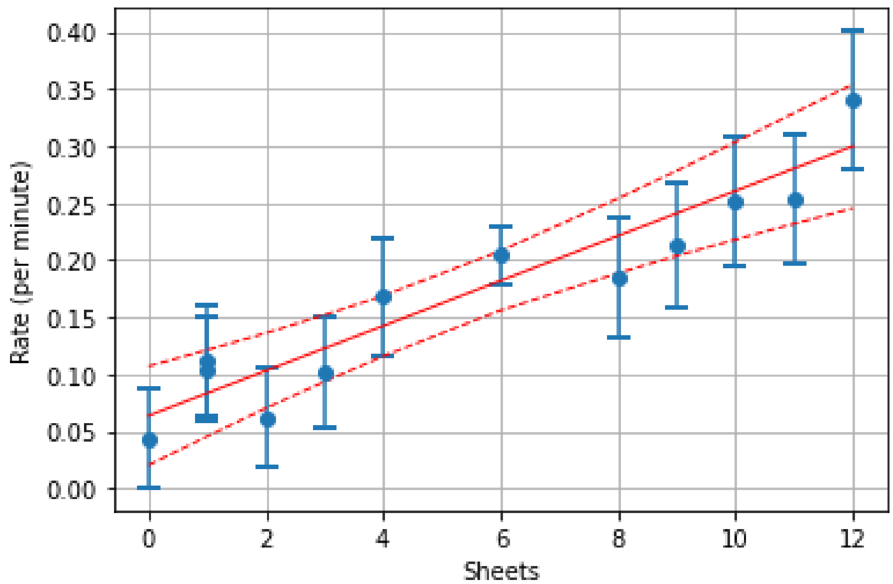


Figure 7: Fitted Data without Sheets 5 and 7

For fig. 7, I removed sheets five and seven as they were most likely caused by higher rates of erroneous signals. This correction give a fit with a slope of $0.019 \pm 0.003\ \frac{1}{\text{min sheet}}$ and a χ^2 per degree of freedom of 0.555.

6 Discussion and Conclusion

6.1 Analysis

My scale was not precise enough to measure the mass of each sheet. The scale I had could measure up to 0.01 g which I thought would be sufficient since that corresponds to a thickness of 0.32 μm. However, fig. 3 shows no measured increase in mass between most sheets applied. For example, sheets 9 through 12 all had a mass of 4.790 g. This means that my scale could not measure the actual change; hence, using eq. 13 the sheets would have zero thickness. Since the sheets have to have some amount of thickness, I plotted all my data with the number of sheets instead. This approach works assuming all sheets are approximately the same thickness.

During my run data collection, I noticed the presence of signals which seemed different from both background and α-particles. These signals seemed to be caused by either static electricity or vibrations affecting my setup. I only noticed these signals after applying the fifth sheet, as there was a major drop in the count rate. This is the reason why I ran the fifth sheet three times. From that sheet forward, I was more careful to control for these signals. I achieved this by spending the first 30 minutes of each run monitoring an oscilloscope to see if a large number of these signals were present. I would also restart runs any time that the building vibrated due to nearby construction. The second run of the fifth sheet and the first run of the ninth sheet are when I did not control properly and had an unusually high number of these signals. Since I only noticed these signals on the fifth sheet, I had to rerun the first four sheets to properly control for them. I still believe variations in these signals might have led to a higher count in the fifth and seventh sheets, so I created two fittings, one with and one without these sheets.

The run data does have a positive trend. In fig 6, the fit has a slope of $0.020 \pm 0.003 \ \frac{1}{\mathrm{min\,sheet}}$ and a χ^2 per degree of freedom of 2.043. This value of χ^2 is greater than 2, indicating that there is some discrepancy between the data and the fit. The largest source of discrepancy is the larger rates measured for sheets five and seven, most likely caused by higher rates of erroneous signals. Therefore, I also graphed my data without these sheets in fig. 7. This graph was fitted with a line of slope $0.019 \pm 0.003 \ \frac{1}{\mathrm{min\,sheet}}$ and has a χ^2 per degree of freedom of 0.555. The lower value means that the data is described well by the fit, and the line falls within the uncertainty of each point. Both of these fits have a positive slope which agrees with the theory (Eq. 8).

6.2 Issues and Future Improvement

In order to achieve better data, there are some improvements that can be made.

1. A base for the gold foil holder. Whenever a new sheet of foil was attached, I would need to take every part of its holder out and then try to reposition it. To improve this, a small piece can be added that holds the plastic that the foil is positioned on. Then the plastic can be slid in and out of the setup which would eliminate the minor variations in it's position.

2. Utilising the analog-to-digital converter (ADC) in the VME crate. This would allow for better signal analysis than the simple counting module as it it can measure integrated signal size and signal shape. Therefore, it would be easier to distinguish between α-particles, background, and the erroneous signals in my data.

3. A more precise scale. This would give better data on the mass of the gold foil and thickness of each sheet. A more precise scale would be harder to use as it would be more sensitive to air currents. Also, some of the foil that was not adhered well would fall off. This happened to the edges of the sheets so it did not effect my scattering data. A more precise scale would pick up these slight changes to the mass of the apparatus.

4. Spending more time characterizing the PMT, α-signals, and background signals. By finding out the relative size of both α-signals and background signals, the equipment can be tuned to help cut out a large amount of the background which would decrease the chance of statistical variation just from those signals. Taking more data on the PMT can help find the optimal voltages for both the power and the discriminator to ensure the largest amount of α-signals are detected while not counting too much background.

6.3 Conclusion

I created an apparatus which is capable of detecting α-particles and collected data measuring the scattering rate's dependence on material thickness. The data was then fitted with two linear fits. The first has a slope of 0.020 ± 0.003 $\frac{1}{\text{min sheet}}$ and a χ^2 per degree of freedom of 2.043. The second was created by removing data points that were most likely errors. It has a slope of 0.019 $\pm$ 0.003 $\frac{1}{\text{min sheet}}$ and a χ^2 per degree of freedom of 0.555. Both fits showed that scattering rate increases with the thickness of the material. This is in agreement with Geiger and Marsden's original result and the Rutherford scattering model (Eq. 8), which indicates that scattering happens within the volume of a material and therefore depends on the number of nuclei present.

7 Bibliography

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H. Geiger and E. Marsden, "On a Diffuse Reflection of the α-Particles.", Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, 1909