Difference Orbit Optimization for Accelerator Optics Correction

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1 Abstract

rayTrace experimentation is a method of producing a "synthetic" beam for the purposes of studying changes in twiss parameters along the CEBAF beamline. The position of the beam transverse to its direction of motion is provided by measurement devices called beam position monitors (BPM) located throughout the system. The progression of the beam can be represented as a series of ellipses characterized by the twiss parameters at each BPM. Many important beam characteristics can be studied by modeling the progression of the beam, as well as used to make globalized optics corrections and optimize beamline tuning.

2 Introduction

The Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Labs is a 12 GeV particle accelerator which utilizes a variety of different detection methods to study beam dynamics. For these purposes, CEBAF can be described as a system of magnets that transport an electron beam from one location to another, much the same way that a system of lenses and prisms transport a ray of light from one location to another. Many measurement devices called beam position monitors (BPM) are located throughout the system, and provide measurements of the beam's position transverse to its direction of motion. The magnets determine how the beam's position, direction, size, and shape will change as it passes through, and the BPMs record these changes for tracking purposes. As a result, it is important to understand how changing each magnet affects the beam, and how severely it does so. To this effect it is the aim of this research to design a system which can determine differences between expected and actual beam trajectory.

3 Theory

The origin and backbone of this research is Ryan Bodenstein's "A Procedure for Beamline Characterization and Tuning in Open-Ended Beamlines". Bodenstein's dissertation has served as the principle resource for information regarding twiss parameters, rayTrace data collection, and phase space. Therefore the following theory excerpts are taken directly from Bodenstein's thesis so as to avoid loss of information from paraphrasing.

3.1 Phase Space

Studying the statistical properties of a collection of particles is both more practical and more useful than studying the properties of a single particle. In order to do this, one investigates the dynamics of the particle in six dimensional phase space, which is represented by the coordinates and momenta, $(x, p_x, y, p_y, \sigma, E)$. Here x and y are positions, $\frac{p_x}{p_0} \approx x'$ and $\frac{p_y}{p_0} \approx y'$ show the transverse momenta, $\frac{cp_0}{\beta} = E_0$ is the ideal particle energy, σ is the coordinate along the trajectory, and E is the particle energy. Alternatively, E is often represented as an energy deviation, or a relative energy deviation with respect to a reference, or ideal particle. Additionally, if beam energy is constant, it is common to use the slope of the trajectories, x' and y', rather than the transverse momenta, as they are proportional to the momenta and also small, thus allowing one to let $\sin(x') \approx x'$.[1]

3.2 Twiss Parameters

Attributes of the beam can be described by the Twiss parameters. These terms describe the area in phase space that is occupied by the particle beam. $\alpha = \frac{-\beta'}{2}$ is related to the distance to the geometric focus from a lens. When it is positive, the beam is convergent, and when it is negative, the beam is divergent. β is an amplitude function, and $\sqrt{\beta\epsilon}$ is the radius of the beam, where ϵ is the emittance. The emittance is proportional to the area that a set of beams will occupy in phase space, as $Area = \pi\epsilon$. γ relates the focusing distance and the amplitude, and is defined as $\gamma \equiv \frac{1+\alpha^2}{\beta}$. These terms are all interrelated, as demonstrated by the phase ellipse in Figure 1. [1]

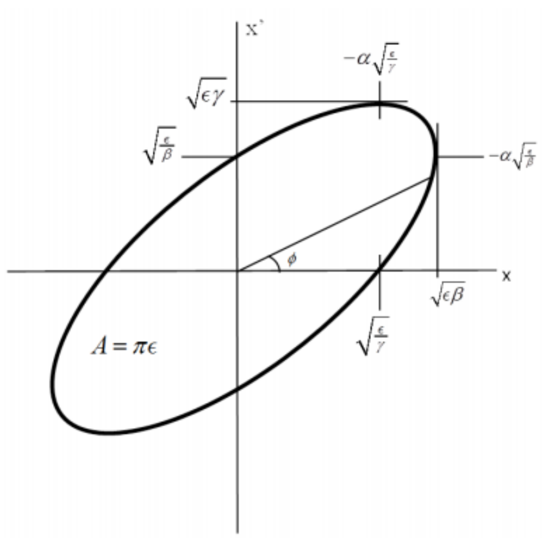

Figure 1: Betatron Ellipse in Phase Space

4 Methods of Strength Generation

RayTrace experimentation aims to study the progression of twiss parameters throughout the beamline by generating a "synthetic" beam. This synthetic beam is produced by generating a series of ellipses in phase space characterized by the twiss parameters at each BPM. The process begins by generating a unit circle with N evenly spaced (X, Y) data points around the origin. Each data point then undergoes an inverse Floquet transformation to change to (X, X') space.

$$\begin{pmatrix} X \\ X' \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix} \begin{pmatrix} \sqrt{\epsilon\beta} & 0 \\ \alpha\sqrt{\frac{\epsilon}{\beta}} & \sqrt{\frac{\epsilon}{\beta}} \end{pmatrix} \tag{1}$$

The following transfer matrix M describes the evolution of the beam from one point to another. It can therefore be used to solve for dx'_1 and dx'_2. These two dx' values represent the strengths of the two kickers required to steer the beam to the (x_{bpm}, x'_{bpm}) coordinate in phase space.

$$M = \begin{pmatrix} \sqrt{\beta_{bpm} * \beta_1} * \sin(\phi_{bpm} - \phi_1) & \sqrt{\beta_{bpm} * \beta_2} * \sin(\phi_{bpm} - \phi_2) \\ \sqrt{\frac{\beta_1}{\beta_{bpm}}} * [\cos(\phi_{bpm} - \phi_1) - \alpha_{bpm} \sin(\phi_{bpm} - \phi_1)] & \sqrt{\frac{\beta_2}{\beta_{bpm}}} * [\cos(\phi_{bpm} - \phi_2) - \alpha_{bpm} \sin(\phi_{bpm} - \phi_2)] \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} dx'_1 \\ dx'_2 \end{pmatrix} = M^{-1} \begin{pmatrix} x_{bpm} \\ x'_{bpm} \end{pmatrix} \tag{3}$$

5 Data Collection

Over the course of this research process, it was unknown whether or not it would be possible to collect physical data from the accelerator itself. In accordance with this, an Accelerator Task List (ATLis) was written in preparation to do so, and those methods are described in section 5.1. However after the ATLis had been written the accelerator shut down for maintenance early. Physical data will still most likely be taken at a later date as a part of a continuation of this research, but will not be done within the time frame of this project. Since physical data could no longer be taken, the final deliverable had to be changed to analysis through Monte Carlo simulation, as described in section 5.2. Both the physical and simulated methods for data collection are described in the following sections.

5.1 Methods of Physical Data Collection

Whenever the continuous electron beam is running at CEBAF, data is being collected at a frequency of 1 Hz. This means that approximately 1 data point (X, Y coordinate pair) is recorded at every single BPM once per second. The data collection for this experiment involves six data files with two kickers specified in the name of the file. Each file contains two columns of kicker strengths. The first column represents the strengths of the first kicker, and the second column represents the strengths of the second kicker. The two specified kickers have their strengths set to the first pair of strengths in the data file, then operations pause for 10 seconds. Since the beam is collecting one data point every one second, pausing for 10 seconds is the same as saying "collect 10 data points". After 10 seconds the two kicker strengths are changed to the next pair of strengths in the data file, and this process is repeated for every strength pair.

After data collection has reached the end of the file, the process returns back to the beginning of each file three times, such that each kicker pair is set to each corresponding strength pair three times. The point of restarting the cycle three times is to try to correct for longevity errors. Collecting 30 continuous data points at one position is not the same as taking 10 data points at 3 separate intervals. This method allows for an average position to be calculated, and identifies possible beam drifting.

5.1.1 Timing

Since the accelerator is not always running, beam time is a finite resource. As a result it is necessary to be cognizant of how long the experiment is expected to take. It is also worth noting that the beam is lost approximately five times per hour. Statistically speaking the beam is lost every 12 minutes, so it is in the best interest of anyone taking data to keep intervals of data collection below 12 minutes. This is done to reduce the probability that the data collection will be corrupted or negatively influenced by the lost beam. To this effect, there are four main choices that determine the approximate amount of time needed for all necessary data to be collected. The first choice is the number of data points in each ellipse. This number needs to be large enough that one could reasonably detect the shape of an ellipse, but not so large as to make the data collection take too long. The second choice is how long to pause for data collection, or how many data points should be collected per pass through the file. Once again this number needs to be large enough that trends can be detected and meaningful data can be extracted after the experiment ends, but not so large as to risk crashing the beam. The third choice is the number of passes or cycles (the number of times to repeat each file). As the number of cycles is increased, the error at each point is reduced, but time increases as well. The final choice is the number of kicker pairs to test. There needs to be a sufficient number of pairs tested in both the vertical and horizontal direction in order to conclusively determine whether any trends are global, or intrinsic to that particular kicker pair. This time relationship is fairly straightforward, and can be represented by the following equation:

$$T = N * D * C * F \tag{4}$$

Where N is the number of data points around the ellipse, D is the delay to collect data, C is the number of cycles, and F is the number of data files (kicker pairs). For this experiment $N = 8, D = 10, C = 3, F = 6$. Eight points around each ellipse is a sufficient enough number that the shape of the ellipse can be made out without too much difficulty, and visual patterns can be recognized in the plots. A 10 second delay at each point is long enough to collect a reasonable number of data points without significant risk of losing the beam. Six data files were chosen to collect data at three horizontal kickers, and three vertical kickers. This should be enough to detect consistent trends on each plane. Finally three cycles through the ellipse is the largest number of repeated passes that can be achieved while reasonably expecting not to lose the beam. These numbers when multiplied give a total experiment time T of 1440 seconds, or 24 minutes. This seems to be a reasonable amount of time for an undergraduate physics student to expect to be able to use a $300 million particle accelerator.

5.2 Simulated Data Collection

In order to generate useful data for further analysis, different sources of error are systematically introduced to the Elegant model. The strengths generated (as described in section 4) are used to generate N different beams which are tracked together to provide useful information. In order to determine if the beam is appearing where it is supposed to be, the X and Y centroid values are tracked at a given BPM, and are either written to a file or plotted in phase space. When plotted, these centroid values should produce a recognizable ellipse which can be tracked throughout the beamline.

However again since Elegant is a "perfect" beamline, there are multiple unaccounted for sources of error which would appear in the actual accelerator. The three main sources of error which were the focus of this experiment were: quadrupole displacement uncertainty, quadrupole strength uncertainty, and BPM measurement uncertainty. Each of these errors are added to the simulation in a different way, and the effects on the integrity of the simulations predictive abilities are studied thereafter.

5.2.1 Quadrupole Displacement Uncertainty

In the real beamline, the actual placement of the quadrupoles relative to one another isn't exactly on design. Variation of the placement of each quadrupole changes the distance between the magnets, and can add error to the physical accelerator that isn't present in the simulation. To account for this the lattice file has two parameters, DX and DY which represent the offset of each quadrupole. A Gaussian error on the order of 300 microns is added to both the DX and DY for every quadrupole and the changes are saved to a modified lattice file. This modified lattice file is used in place of the normal one and the beam is simulated with the displacement errors.

5.2.2 Quadrupole Strength Uncertainty

The strength of each quadrupole is only known to a certain level of confidence in the actual accelerator, so quadrupole strength errors have to be simulated as a percentage of the design strength. The process for adding quadrupole strength uncertainty involves following the normal simulation procedure up to the point where each quadrupole strength is generated. Once the strengths data file has been populated, each strength value has a randomly generated Gaussian value added to it, with a standard deviation of 5% the original strength. These modified strengths are then used to generate the centroid values which are further processed for chi squared per degree of freedom calculations.

5.2.3 BPM Measurement Uncertainty

BPM measurement uncertainty is the only source of error that is added after the simulated measurements have finished. The simulation assumes that the BPM's can measure the centroid positions of the beam in the X and Y direction with absolute certainty, when in reality there is some error. Gaussian error on the order of 100 microns is added to the centroid values in both the x and y direction (CX, CY) to account for this uncertainty. The new centroid values are saved to a modified data file and are passed forward to be plotted and used for chi squared per degree of freedom calculations.

6 Monte Carlo Simulation

One of the greatest strengths of a Monte Carlo simulation is that it allows for determination of acceptable error thresholds. To determine the tolerances of each error source, 1000 hypothetical beamlines are generated using Elegant, and the difference between the perfect simulation and the "flawed" simulations are determined using chi squared per degree of freedom calculations. The average chi squared per degree of freedom value is then calculated for these 1000 trials, and final conclusions can be drawn.

7 Results and Conclusion

In an ideal Monte Carlo simulation, one perfect simulated beamline is generated, then compared to upwards of 1000 beamlines with different errors included. However, each simulation is still subject to the time limitations discussed in section 5.1.1. Generating 1000 beamlines with 16 points each, for three different error sources would take close to 100 hours of computation time. This would be an ideal number to reduce statistical error, however running the simulation for this long makes it prone to crashing. In order to minimize these risks, 300 beamlines of each error type with 16 orbits each were generated, requiring only 10 hours of total computation time and providing some initial quantitative data. The measure of how close the perfect beamline was to the error filled beamline was done using a chi squared per degree of freedom evaluation of the normalized orbits. The results of these simulations are shown in the table below.

Type of Error	Strength of Error	Chi Squared per Degree of Freedom
Strength Uncertainty	5%	1.37156
BPM Measurement Uncertainty	0.001	6.98311
Displacement Uncertainty	0.003	1.65142 e-6

Each chi squared per degree of freedom (CHI2DOF) value in the table above allows for a different conclusion to be drawn. For the quadrupole strength uncertainty, the CHI2DOF gives indication that a tolerance of 5% is most likely acceptable for the actual accelerator design. This CHI2DOF was the second largest of the three, meaning that it is the second most influential source of error. However, since the value is very close to one it is still not expected to have completely destructive effects on the outcome of the simulation. The CHI2DOF value for bpm measurement error was above the generally accepted value of 1, but not by very much. This indicates that the tolerance for this source of error is possibly an order of magnitude too high. The CHI2DOF value is not so large as to be indicative of an implementation error, but it is still slightly above what is considered acceptable tolerances. Finally the quadrupole displacement uncertainty was by far the smallest CHI2DOF value, indicating that this error is the least influential. The value is almost zero, meaning that the placement of the quadrupoles has virtually no effect on their ability to accurately steer the beam to design positions.

8 Future Work

The original intention of this work was to compare the simulated rayTrace data with physical data collected in the accelerator. One of the next stages of testing this method of globalized optics corrections is comparing the simulated betatron ellipses to those constructed using physical data. Also, the initial Monte Carlo simulations only involved generating 300 simulated beamlines to determine an average chi squared per degree of freedom value. Another next stage would be to generate the full 1000 simulated beamlines to reduce statistical error. However the current script still takes too long to run, and is prone to crashing when running for extended periods of time. Therefore future work would involve optimizing the simulation code to reduce run time. Finally, this entire project involved studying just one type of optics identification (rayTrace), so future work would involve developing a second method for comparison purposes. Possibly creating a method that tracks the transformation of individual particles rather than points around a design ellipse.

References

[1] M Bodenstein, Ryan. *A Procedure for Beamline Characterization and Tuning in Open-Ended Beamlines.* University of Virginia, 2012.