Measuring the Speed of Light:

Investigating Fizeau's Method

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In the nineteenth century physics was a prospering field with discoveries and advancements made all across the board in the topics of electricity, magnetism, optics and thermodynamics. In the mid nineteenth century physicists began to make the connection between light and to electromagnetism. It was not until 1861 that Scottish physicist James Clerk Maxwell had put together a set of four equations describing the nature of electric and magnetic fields, now known as Maxwell's Equations. Using these four equations, Maxwell derived that light was in fact an electromagnetic wave propagating at approximately 300,000,000 meters per second. About a decade before Maxwell had made this connection and before the theoretical speed of light was derived, French physicist Armand Hippolyte Louis Fizeau had experimentally revealed a measurement of the speed of light to be around 313,274,000 meters per second, a mere 4% difference from the currently accepted value. (O'Conner and Robertson)

Armand Fizeau was a physicist who was widely known for reconfirming other experimental results or trying to pick up where other experiments had failed. Fizeau had previously read about Italian physicist Galileo Galilei, who in 1638 predicted that light had a speed that could be measured. Galileo's method for doing this involved arranging himself and an assistant with lanterns on two separate hill tops. One lantern would shine, signaling the other person to shine their lantern. By measuring the delay in the lanterns' illumination, Galileo would measure the speed of light. As one might imagine, the human reaction time proved to be incapable of such precision and Galileo failed to record a speed but he postulated that the speed of light was ten times faster than the speed of sound. Fizeau however, was inspired by Galileo's attempt and his prediction of a definite speed of light and created a remarkably simple experiment capable of exposing such a measurement. (O'Conner and Robertson)

Fizeau sought out to rectify Galileo's experiment and to show experimentally that light indeed have a speed. In fact, prior to the seventeenth century the majority of scientists, mathematicians, and philosophers believed the speed of light to be infinite. Johannes Kepler and Rene Descartes are two famous scientists who believed light to have infinite speed. Fizeau, however, along with most physicist during the nineteenth century, predicted light speed to be finite, and Fizeau wanted to know if he could succeed where Galileo failed. In order to do this, Fizeau created an ingenious apparatus involving a toothed wheel, some mirrors, and of course, a light source (Setterfield, Barry).

For this capstone project, the goal was to understand, recreate, and improve Fizeau's original apparatus. I also wanted to accurately (within 10%) measure the speed of light using the device. A clear understanding of the apparatus needed to be obtained, as well as a range of values for the parameters of the experiment. The equipment utilized and purchased needed to be both capable of achieving these goals and staying within the project budget of $200. To organize these steps, a tentative schedule was formulated to stay on track and make timely progress in an attempt to complete the project in the allotted time frame.

The first question to consider is: what *was* Fizeau’s experiment? Fizeau’s experiment involved a toothed wheel, a series of lenses, a light source, a partially reflective mirror, and a near-perfect mirror. The light source and partially reflective mirror would be placed behind the toothed wheel. The light would shine through the partially reflective mirror, only allowing a fraction of the light through to the toothed wheel, which would be rotating at a constant angular rate. The toothed wheel chops up the beam of light into pulses which continue forward to the near-perfect mirror placed at a known distance away from the wheel. The light pulses will reflect back to the toothed wheel, and one of two things will happen: either the wheel will have rotated enough so that the next tooth blocks the reflected light, or the light will come back before the wheel has rotated and will continue through to the partially reflective mirror to be observed.

Throughout the setup the lenses would be placed to focus and collimate the light into a beam. The goal was to have the wheel rotate fast enough to block the incoming light however. Knowing the angular rate of the toothed wheel, the distance between the wheel and near-perfect mirror, and the number of teeth in the wheel, the speed of the light pulses can be formulated (H. Fizeau).

Figure 1 below illustrates the original setup of Fizeau's apparatus.

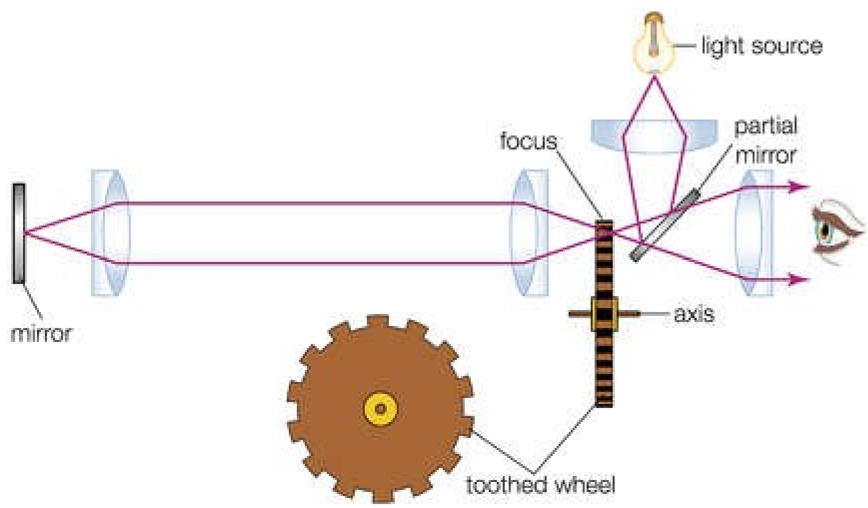


FIGURE 1 (Encyclopedia Britannica)

To understand why this apparatus measures the speed of light, the primary equation this experiment is centered around was derived. This derivation begins with the simple equation for average velocity of an object.

Assuming that the speed of light is a constant (which is known to be true):

$$v = d/t \text{ m/s}$$

Where v is the speed of the light pulse, d is the distance, and t is the time. Let L be the distance between the toothed wheel and the distant mirror. Then the distance d in the equation will be substituted with 2L, since the light must make a round trip. The time defines how long it takes the wheel to switch from a tooth to a gap at the same point. To get this time, the fact that frequency and time are inverses of each other was exploited.

$$f = 1/t \ \mathrm{s}^{-1}$$

If n is the number of teeth and ω *is* the angular frequency of the wheel, then the teeth frequency is given by

$$f = \omega n = 1/t \ \mathrm{s}^{-1}$$

The inverse of this gives how fast the wheel switches from tooth to tooth, but one needs to know how fast the wheel switches from tooth to gap. Taking half of this time, or doubling the frequency gives:

$$2f = 2\omega n = 1/t \ \mathrm{s}^{-1}$$

Rearranging and substituting this time into the original velocity equation yields:

$$v = 4L\omega n \quad \mathrm{m/s}$$

This is the principle equation describing how Fizeau's apparatus works (Serway and Jewett.). It is important to note that this is the equation for the light to be blocked by the *next* tooth. Of course the light could be clocked by the second or third next tooth, or even make a full rotation before being blocked by a tooth. For simplicity I did not derive or look into how the equation would change if I wanted the light blocked by a different tooth .

The most important piece of the experiment is arguably the toothed wheel. Fizeau requested a wooden wheel with 720 teeth from his master craftsman. Using clockwork, he was able to spin his wheel at 12.6 rotations per second, or 756 rotations per minute. The light source that Fizeau used was a lantern, which as one may imagine, is naturally divergent and the light itself is uncollimated. In order to collimate the light and focus the light into a beam, Fizeau made use of some focusing lenses. The distance used had to be very long. Fizeau set up his reflecting mirror approximately 8,633 meters away from the toothed wheel, so that light pulses made a 17,266 meter round trip.

Plugging in the original parameters into equation, Fizeau's result appear:

$$v = 4\mathrm{L}\omega n$$

$$= 4(8{,}633\text{ m})(720\text{ teeth})(12.6\text{ rotations per second})$$

$$= 313{,}274,\ 304\text{ m/s}$$

In order to recreate the experiment, some changes needed to be made in the equipment and the parameters. The first and major change in the modern adaptation for this project was to use an electric motor to rotate the toothed wheel, rather than making use of clock work or a hand crank system. This was done in order to get a more consistent angular speed and to simplify the overall process. Secondly, a neon laser was implemented instead of a lantern. Using a laser provided a light source that was collimated and focused into a beam. A corner cube retro reflector (see the equipment section at the end) would also replace the near-perfect mirror to further simplify the experiment, and the partially reflective mirror would be removed altogether. The retro reflector can redirect the beam of light 180˚ and parallel from the source and thus eliminates the complication of aligning a mirror to reflect precisely back at the incident angle. Figure 2 illustrates the modern experimental setup used in this recreation.

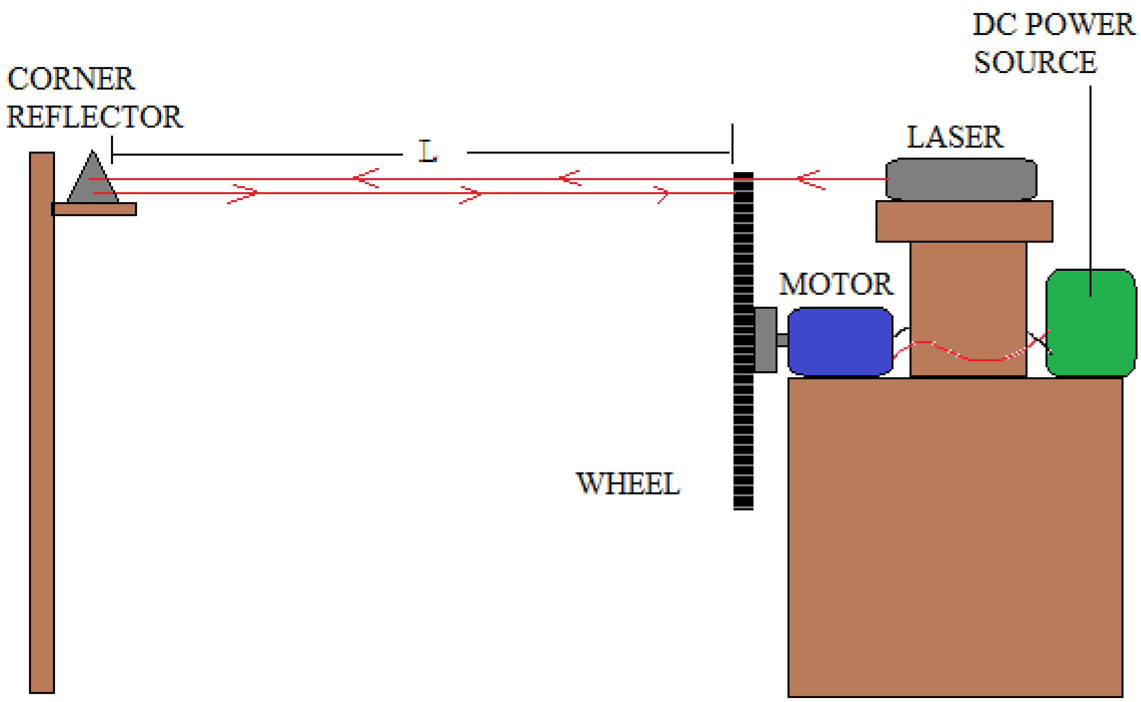


FIGURE 2.

Practically the distance of 8 kilometers needed to be cut down, and the other parameters needed to be increased accordingly. The number of teeth in the original experiment was also a major crafting concern and would need to be adjusted. It was decided that the most practical range would be within 200 of the original 720 teeth. The final parameter to be modified was the rotational rate of the toothed wheel. With modern technology surely the rotational rate could potentially be increased significantly. Using a spreadsheet, I programmed the light speed function in and adjusted the values until I found some combinations that were found to work well. After compromising on a distance of 1000 meters, a range of feasible parameters for the rotational rate and number of teeth was generated. Figure 3 shows a table of values for possible combinations of parameters.

Number of Teeth	Rotational Rate (RPM)	RPS (RPM/60)	Distance (m)	Measured Speed of Light (m/s)	Percent Difference from Original
500	9000	150.00	1000	300000000.00	4.24
550	8200	136.67	1000	300666666.67	4.02
600	7500	125.00	1000	300000000.00	4.24
650	6900	115.00	1000	299000000.00	4.56
700	6400	106.67	1000	298666666.67	4.66
720	756	12.60	8633	313274304.00	0.00
750	6000	100.00	1000	300000000.00	4.24
800	5600	93.33	1000	298666666.67	4.66
850	5300	88.33	1000	300333333.33	4.13
900	5000	83.33	1000	300000000.00	4.24
950	4700	78.33	1000	297666666.67	4.98
1000	4500	75.00	1000	300000000.00	4.24

FIGURE 3

To specify a motor, specifications such as the output power and torque were taken into account. The basic equations for torque were manipulated to see how long the motor would take to get the wheel up to proper speed.

Starting with the basic equation relating power and torque:

$$P = \tau \omega$$

Rearranging and substituting the definition of angular frequency yields

$$\tau = I\,\alpha = \frac{P}{2\pi\,f}$$

where f is rotations per second. The relationship for the time it takes to get the motor to max speed is:

$$\tau\,t = I\,\omega = 2\,I\,\pi\,f = mR^2\pi\,f$$

Solving for t yields:

$$t = \frac{mR^2\pi\,f}{\tau}$$

Substituting our value for torque above gives:

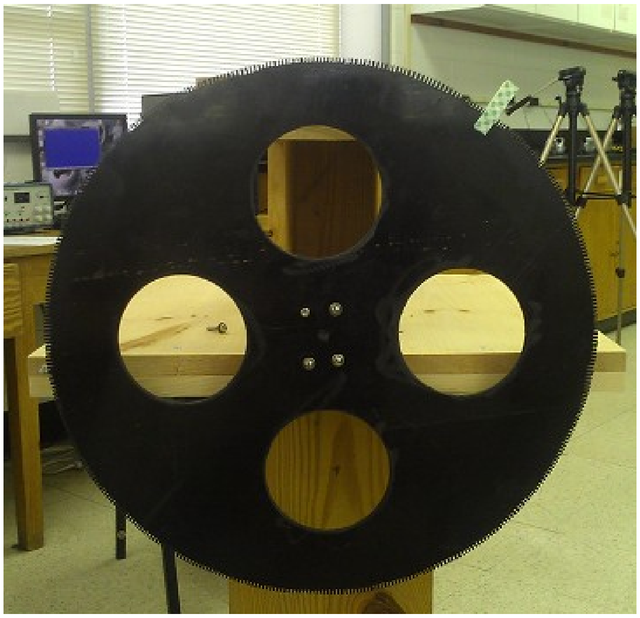
$$t = \frac{2\mathrm{mR}^2\,\pi^2\,f^2}{P}$$

This is the equation used to determine how long it would take a given motor to get the wheel up to its max rotational rate. Substituting values for a wheel with a mass m of about 1.5 kilograms and radius R of 0.25 meters, a motor power of 133 watts, and a max rotational rate of 8000 rotations per minute (or about 133 rotations per second) the motor would take approximately 370 seconds (or just over 6 minutes) to get the wheel up to speed.

$$t = \frac{2(1.5)(.25)^2(\pi)^2(133)^2}{133} = 369.18\,seconds$$

A used Johnson motor that was advertised as being high torque and could produce 8000 rotations per minute at max efficiency, was chosen to be used. This motor matched the parameters at a price of about $25. Most of the motors looked at that met the parameters ended up ranging from $150 to $500, so finding a motor evolved into a budget constraint issue. After getting the motor, the apparatus was constructed. Using power tools and lumber I made various stands for optical devices, motor and wheel, and the laser. The height of all the apparatus needed to allow the laser, wheel teeth and optical devices to be aligned correctly at the same height.

The wheel used was half a meter in diameter and about a quarter of an inch thick, made from a plastic shatter-proof material called lexan. The wheel was constructed by measuring and cutting a circle from a square plate of lexan then, using a file and a making use of a meter stick, individually filing the teeth into the edge of the wheel. In order make the teeth uniform, a tool used to make gears called a pitch gauge was used. This tool gave the measurements for gear teeth depending on the circumference of the wheel and the number of teeth in the wheel. Photos of the stand made for the motor and the wheel, as well as the wheel are below (more photos in the equipment section at the end).



The wheel was attached to the motor by means of a clamp design that was made in a machine shop from a block of aluminum. The block was cut in half horizontally, and the inside was cut out to shape and fit to the motor shaft. Holes were bored into the the two halves so that screws could be placed to connect the two halves back together around the motor shaft. Four more holes were bore in the block as well as the wheel so that the wheel could be screwed into the block. Figure 4 is a diagram of the basic clamp design.

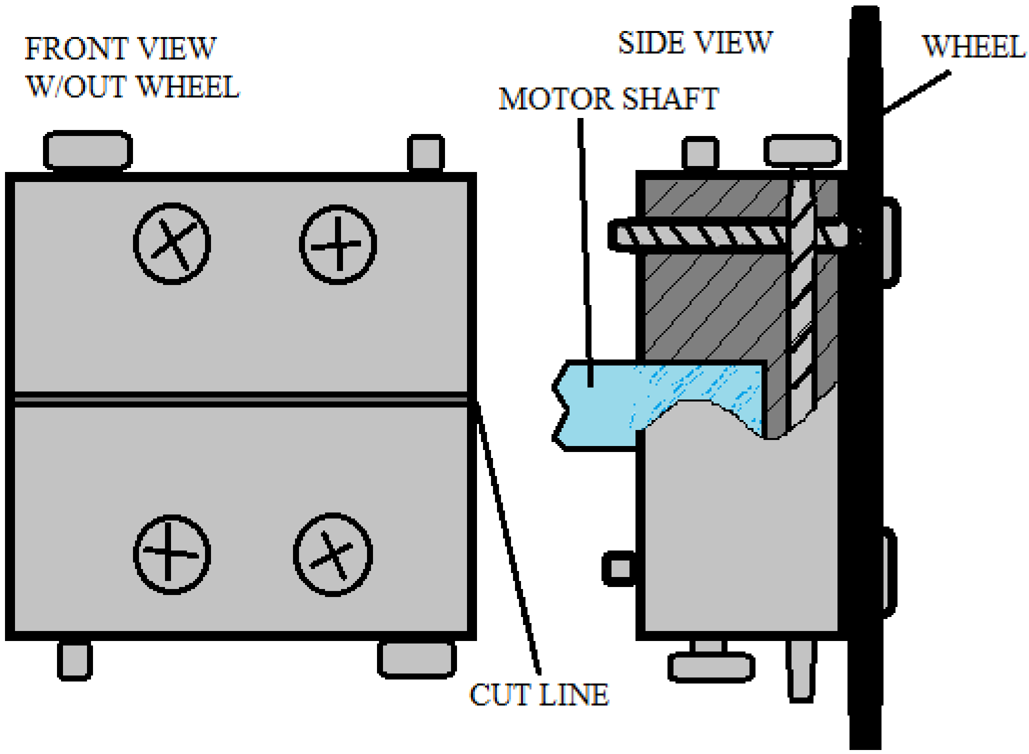


FIGURE 4

After the setup was constructed, the testing phase began. Before running the apparatus it was necessary to make sure the motor was doing its proper job and giving the required frequency. Double sided tape was placed onto the edge of the wheel and a photo gate, provided by the PCSE department, was put over the wheel. The pulse function of the photo gate was utilized to determine e number of rotations per second. According to the speed of light equation derived, using 600 teeth and a distance of 1000 meters meant 133 rotations per second needed to be obtained, but unfortunately the setup yielded slightly more than 5. Testing of the rotational rate was done approximately ten times with the same results each time. The tape passed once every 0.18 seconds which yields approximately 333 rotations per minutes rather than the 8000 needed.

There were a few potential reasons as to why the apparatus was not rotating at the needed rate. The first and most likely cause was that my wheel may have been too heavy and that my previous calculations may have been fundamentally flawed. A second potential cause could have been that my power source was flawed, although this is unlikely. The power source used was a direct current power source provided by the PCSE department.

Another problem encountered came from the actual wheel. While testing for alignment of the laser and teeth it was found that roughly one fifth of the wheel teeth (all in one consecutive section) were not aligned and the laser was shooting over the teeth rather than through or onto them. Although this is a problem it is important to note that the experiment could still have been completed, with a decrease in light intensity during the experiment. One would see flashes of the reflected light one fifth of the time and a significant decrease four fifths of the time. Seeing this decrease would be more difficult, so a light meter may be necessary to be implemented.

To troubleshoot these problems, the wheel was going to be detached from the motor and the motor was going to be quality checked to match specifications provided by the manufacturer. If the motor met specifications a new wheel would possibly need to be made. A new wheel would need to be much lighter, but before *any* troubleshooting could be done, a major problem detaching the wheel and motor was encountered. When a screw connecting the wheel to the clamp became stuck and could not be removed and a head from one of the screws holding the clamp together broke, leaving the clamp unable to be taken off, (at least unable with the tools I had access to) there was no way to detach the wheel and motor. Therefore troubleshooting was never actually initiated.

Adding additional optical devices could make the project simpler. In order to cut out large physical displacements to satisfy the distance parameter, there are a few options that could be explored. One could make use of penta-prisms or additional corner reflectors. A penta-prism is an optical device similar to a corner reflector but rather than redirecting a beam at 180˚, it redirects a beam at 90˚. A design change could implement a few of these devices in series to bend the light path and allow for the distance to be maximized over a minimized displacement. Another option is to utilize fiber optic cable. If a kilometer of fiber optic cable could be obtained and coiled around a spool, it may be possible to direct the beam into the cable and place the corner reflector on the other end. The beam would then be redirected back into the cable. If this route were to be pursued, cable length calculations would need to be considered.

I believe significant progress on understanding how the device works and quantifying feasible parameters for the device was made. I also believe I've provided a foundation and some decent designs for a modern adaptation of the original apparatus. If this project were revisited, with the information I have provided as well as some of the equipment I purchased and built, I believe this project could be successfully continued.

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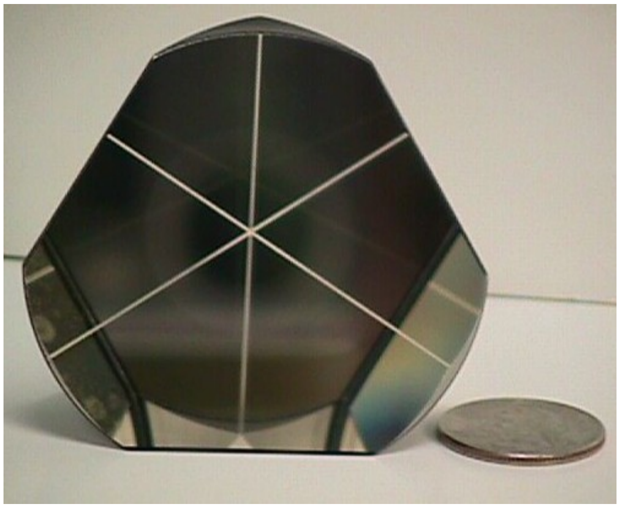
Equipment Used:

Cubic Corner Reflector:



The corner reflector purchased from ebay.com user, mi-laser

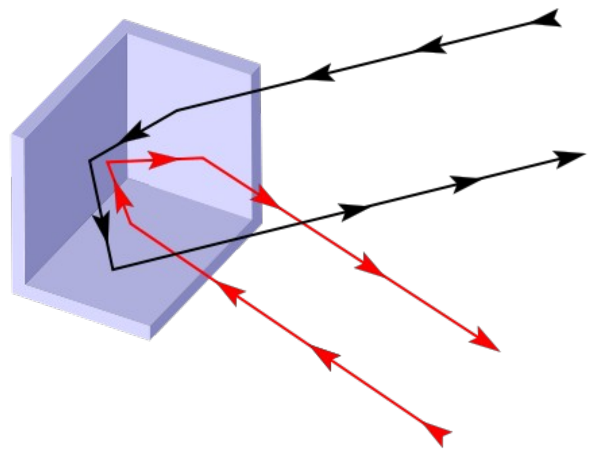
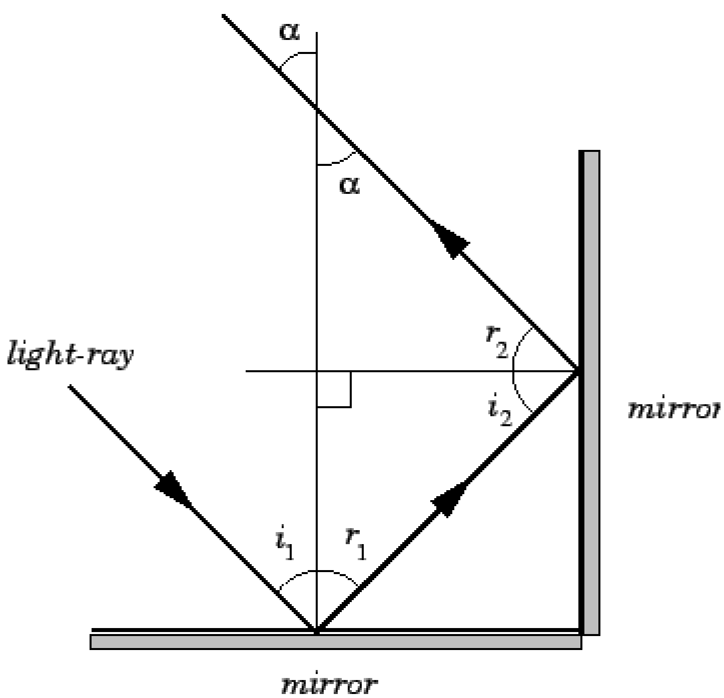


Illustration of how a corner reflector works from http://en.wikipedia.org/wiki/Corner_reflector



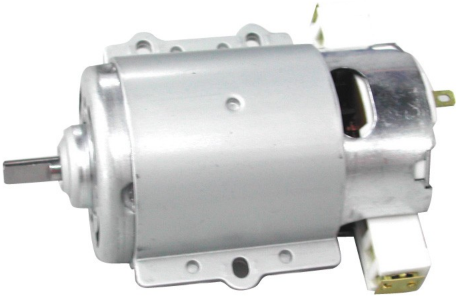
Another diagram depicting how the corner reflector works from http://farside.ph.utexas.edu/teaching/302l/lectures/node133.html.

<u>Constructed Wheel:</u>

Photo of the 600 teeth wheel made from lexan

The wheel and motor attached

A side view of the apparatus



Characteristics:

High Power HVDC Motor

Specifications:

Dimensions	: Ø 52.4 X 116.0 mm
Shaft Diameter	: Ø 6.350 mm
Input Voltage	: 120.0 V DC , Rectified , 60 Hz
No Load Speed	: 11000 rpm
No Load Current	: 0.50 A
Maximum Efficiency	: 64 %
Torque at Maximum Efficiency	: 155.00 mNm
Speed at Maximum Efficiency	: 8200 rpm
Life (typical)	: 500 hr
Weight	: 795 g
Operation Temperature	: -10 to 55 °C
Storage Temperature	: -20 to 80 °C
Electrical Connection	: Terminals
Certification	: UL, cUL

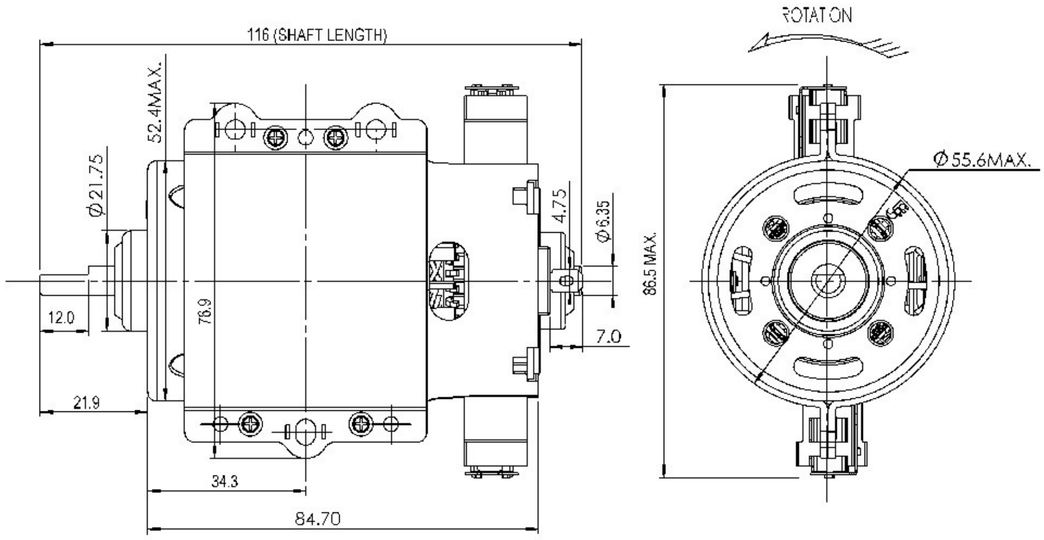
Performance Data:

	No Load	Max Efficiency
Current (A)	0.50	2.16
Efficiency (%)	-	64
Output Power (W)	-	133.00
Speed (rpm)	11000	8350
Torque (mNm)	-	160.00

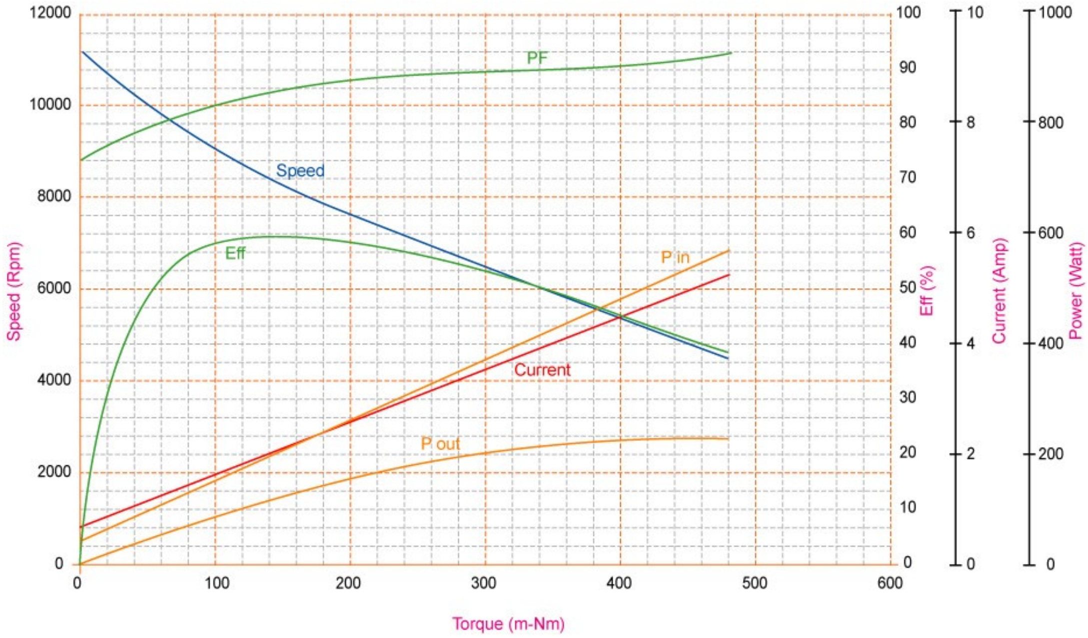
Application Examples:

Floor Care, Power Brushes, Floor Polishers, Brush Roll, Carpet Cleaners

Drawing:



Performance Curves:



Units in Metric